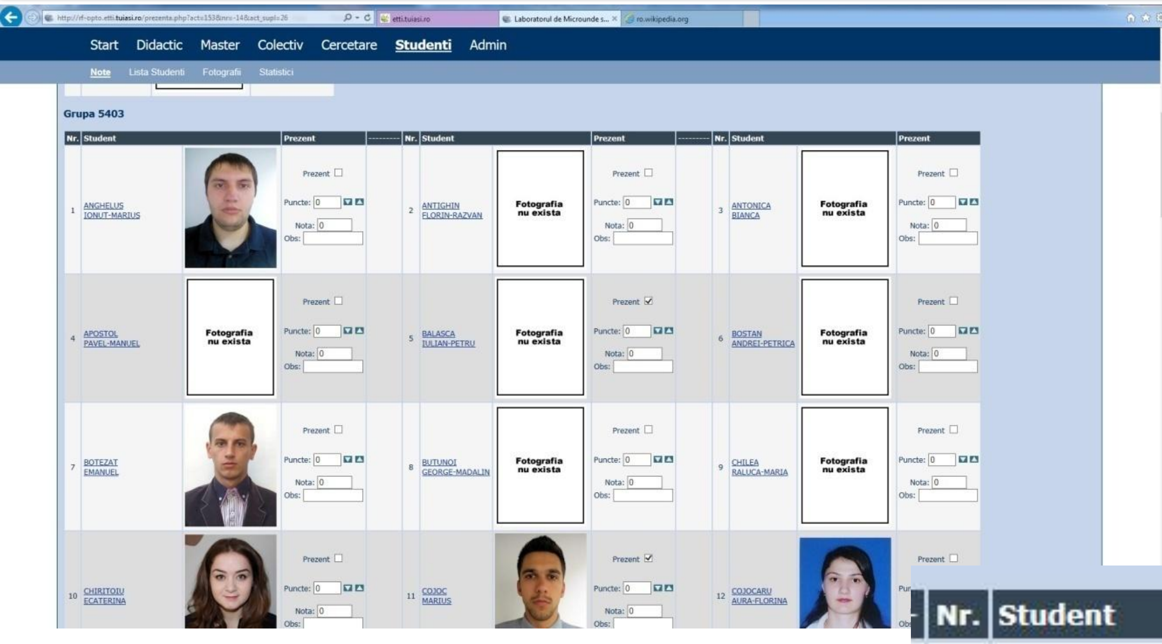
Lecture 4
2017/2018

Microwave Devices and Circuits for Radiocommunications

Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by email: rdamian@etti.tuiasi.ro
  - used at lectures/laboratory

Photos


Start
Didactic
Master
Colectiv
Cercetare
Studenti
Admin
Note
Lista Studenti
Fotografii
Statistici
Grupa 5403
Nr.
Student
Prezent
1 ANGHELUS IONUT-MARIUS
2 ANTIGHIN FLORIN-RAZVAN
3 ANTONICA BIANCA
4 APOSTOL PAVEL-MANUEL
5 BALASCA IULIAN-PETRU
6 BOSTAN ANDREI-PETRICA
7 BOTEZAT EMANUEL
8 BUTUNOI GEORGE-MADALIN
9 CHILEA RALUCA-MARIA
10 CHIRITOIU ECATERINA
11 COJOC MARIUS
12 COJOCARU AURA-FLORINA
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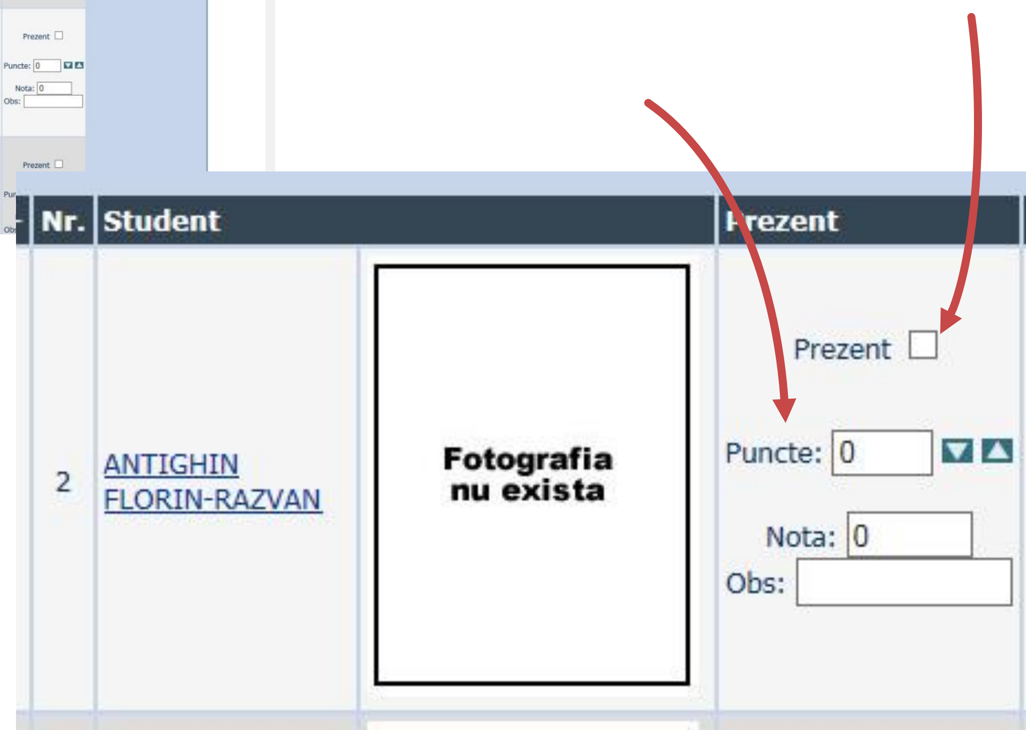
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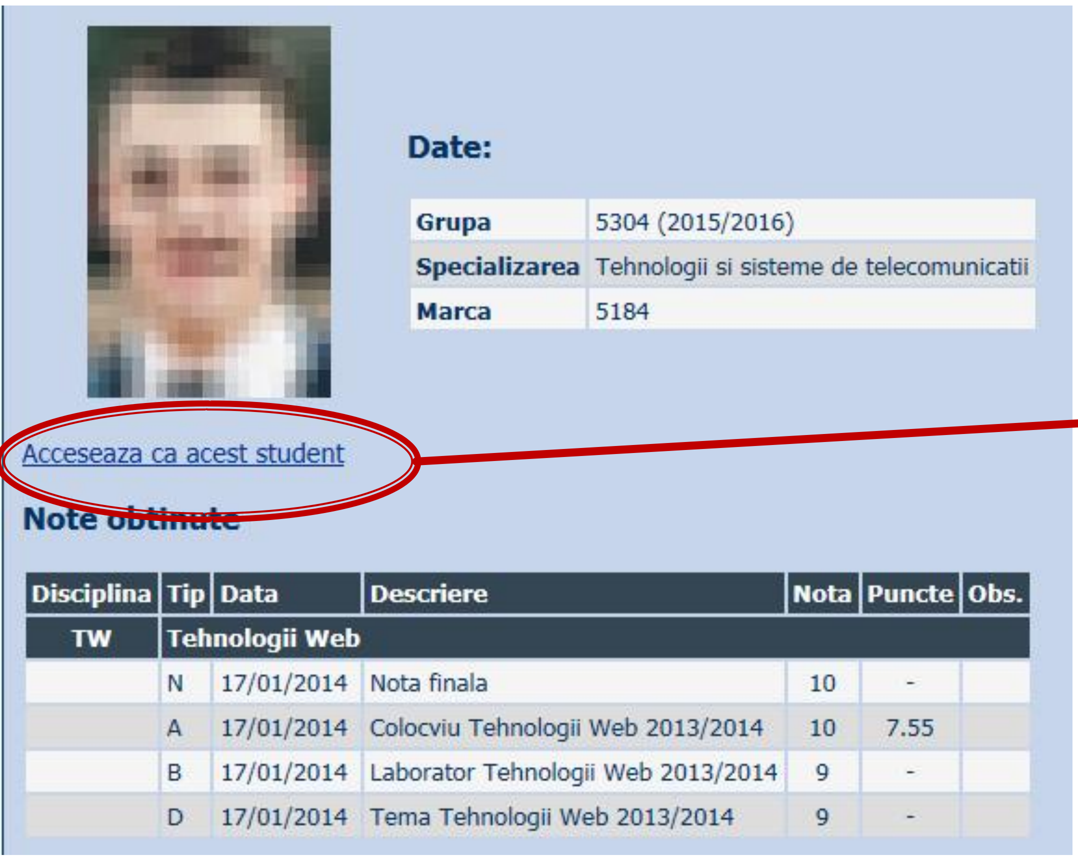
Access

- ~~Not customized~~



Date:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Acceseaza ca acest student

Note obtinute

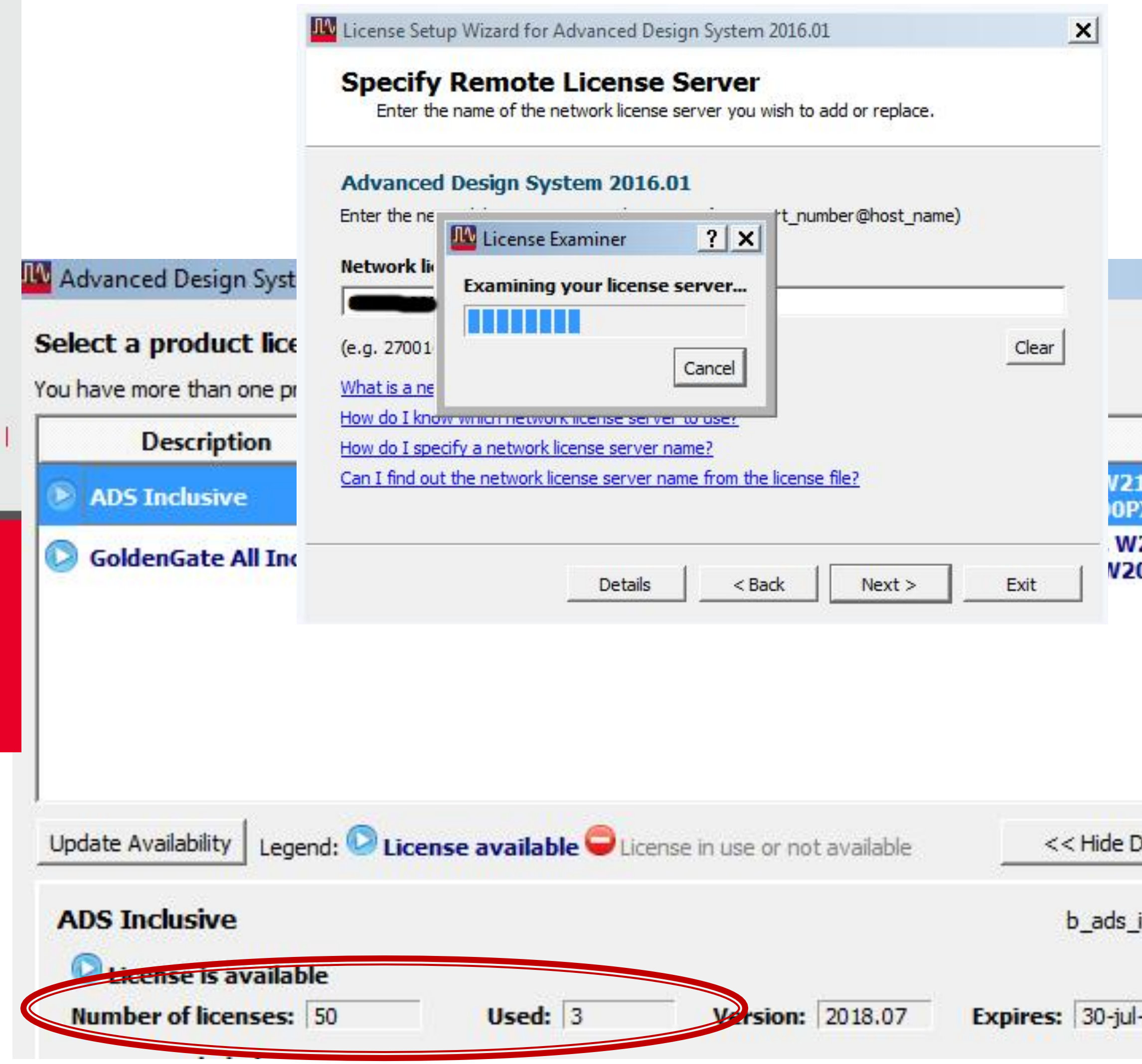
Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

Software

- ADS 2016
- EmPro 2015
- based on IP from outside university or campus

Software

Advanced Design System
Premier High-Frequency and High Speed Design Platform
2016.01
KEYSIGHT TECHNOLOGIES
© Keysight Technologies 1985-2016



License Setup Wizard for Advanced Design System 2016.01
Specify Remote License Server
Enter the name of the network license server you wish to add or replace.
Advanced Design System 2016.01
License Examiner
Examining your license server...
Cancel
Clear
How do I specify a network license server name?
Can I find out the network license server name from the license file?
Details
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Next >
Exit
Select a product lice
ADS Inclusive
Update Availability
Legend:
License available
License in use or not available
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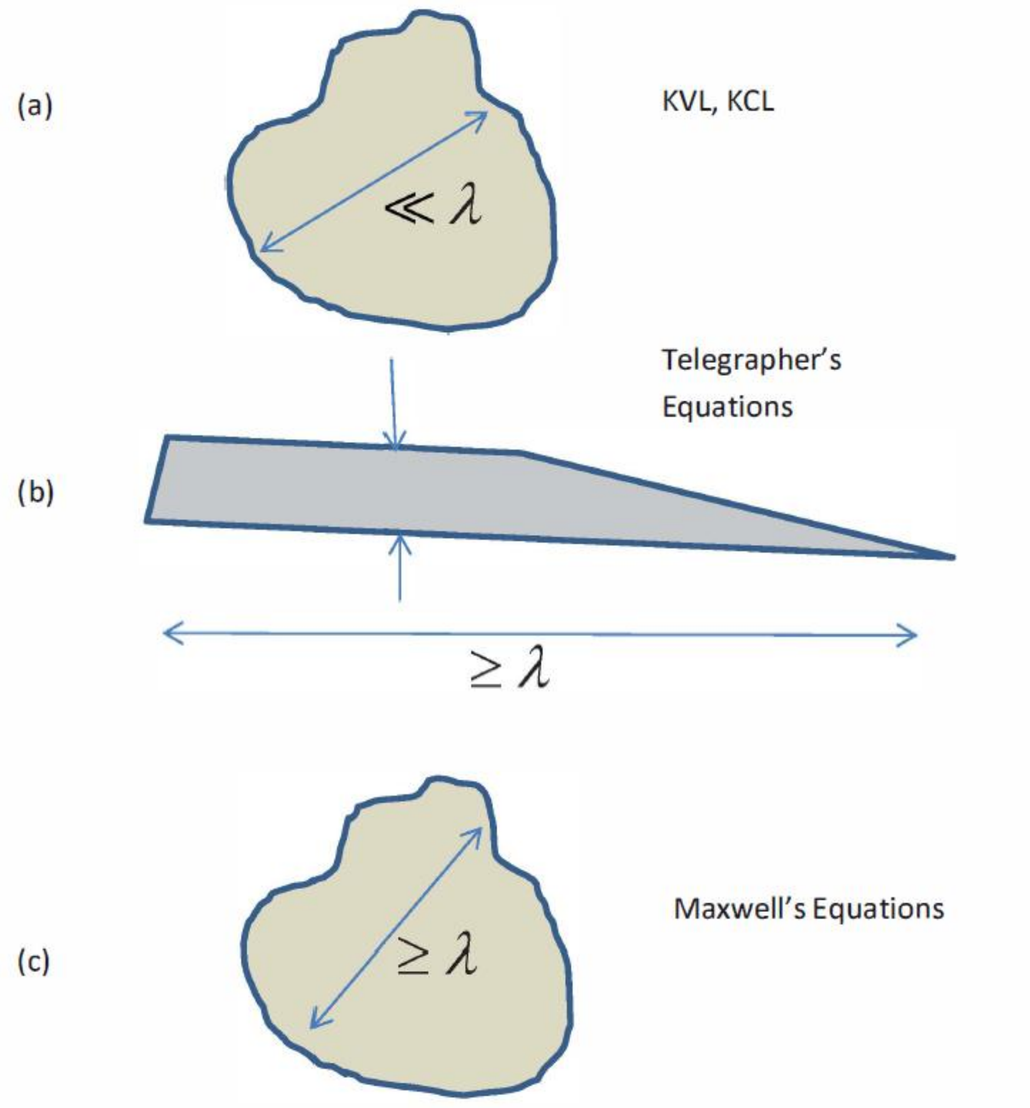
Number of licenses: 50
Used: 3
Version: 2018.07

Electrical Length

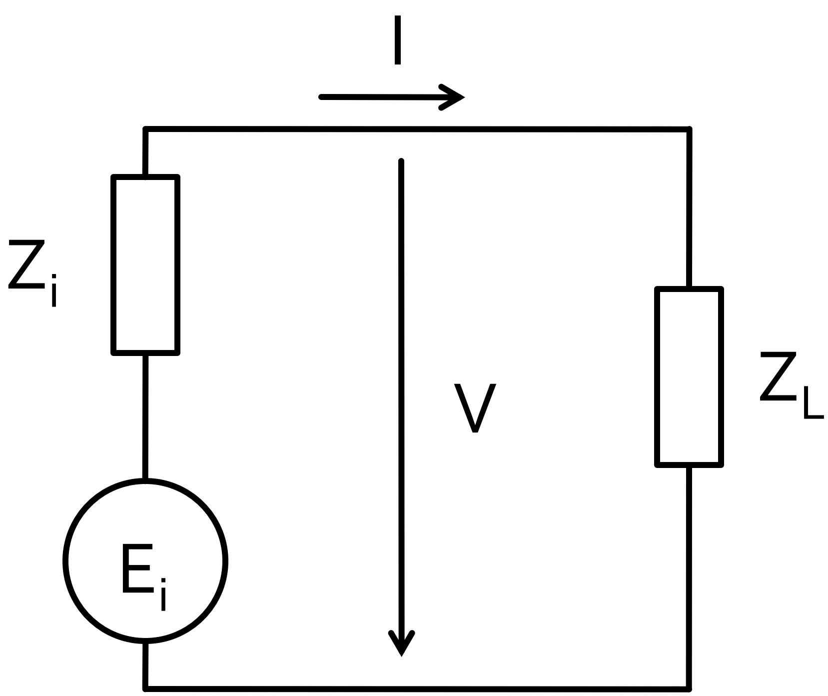
- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest
  - E≈0 → Kirchhoff
  - E>0 → wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



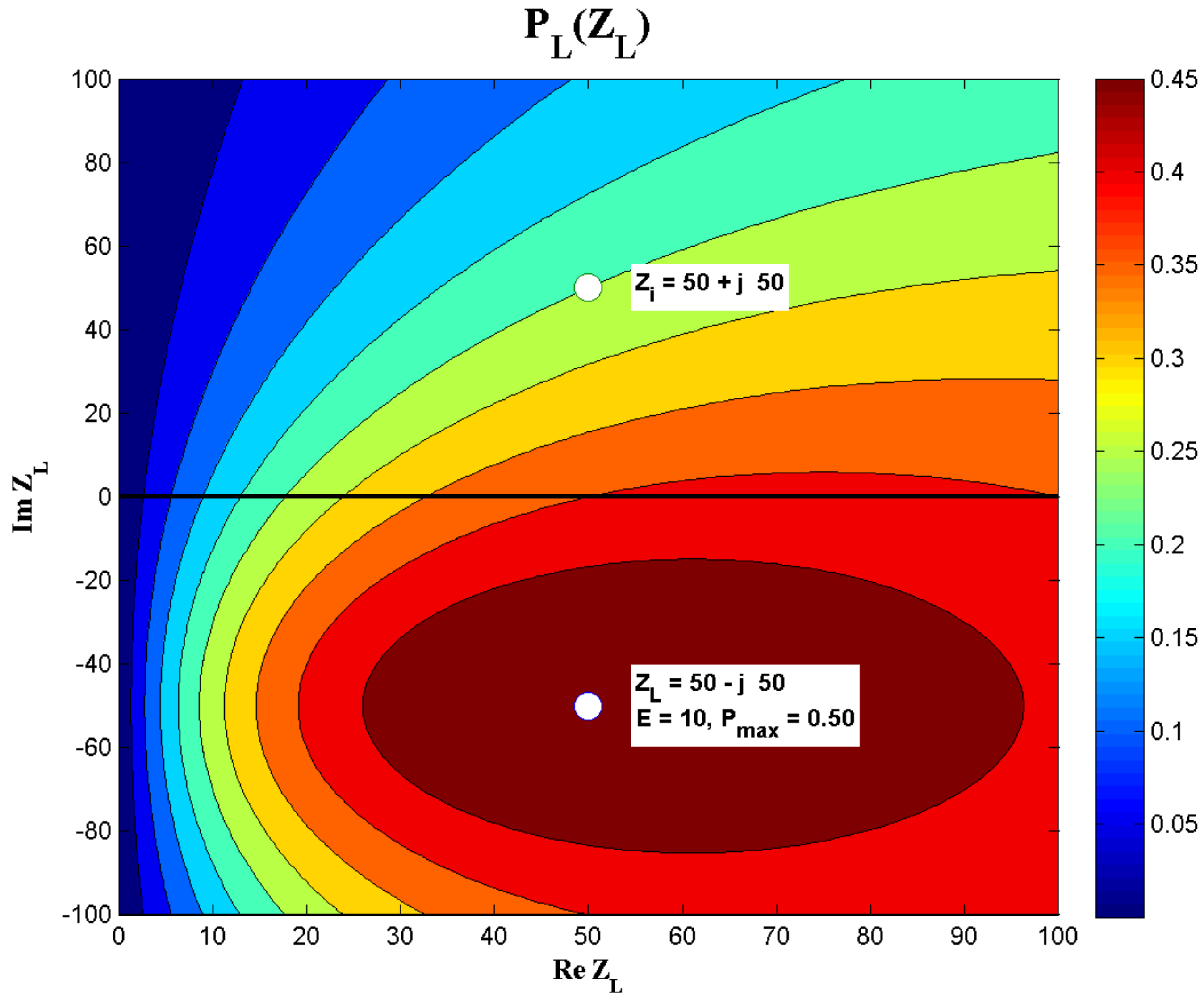
Matching

- Source **matched** to load ?



- impedance values ?
- existence of reflections ?

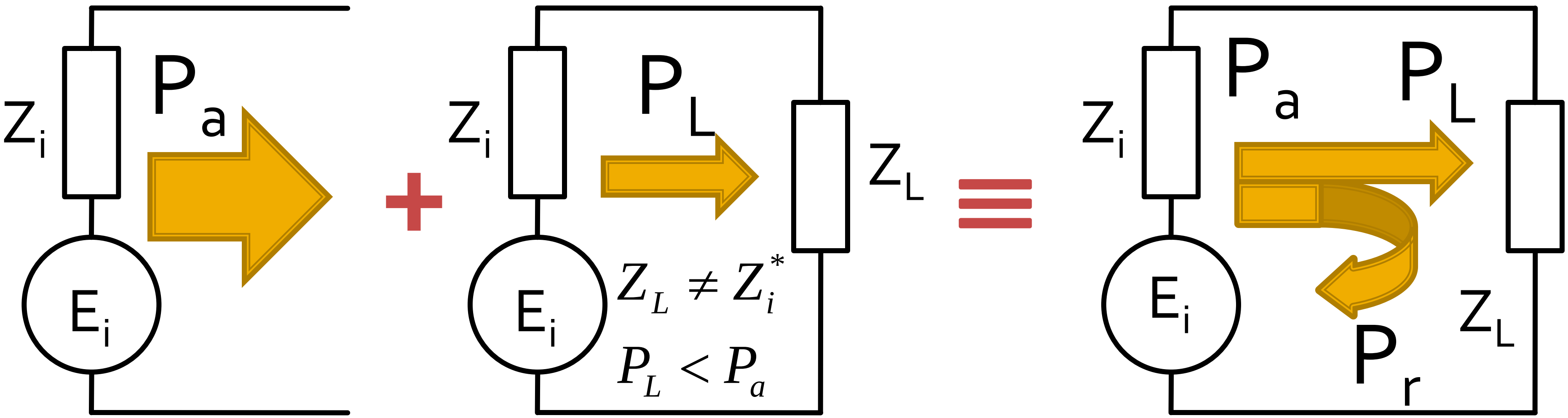
Matching, example



$$Z_L = Z_i^*$$

$$\Gamma_L = \Gamma_i^*$$

Reflection and power / Model

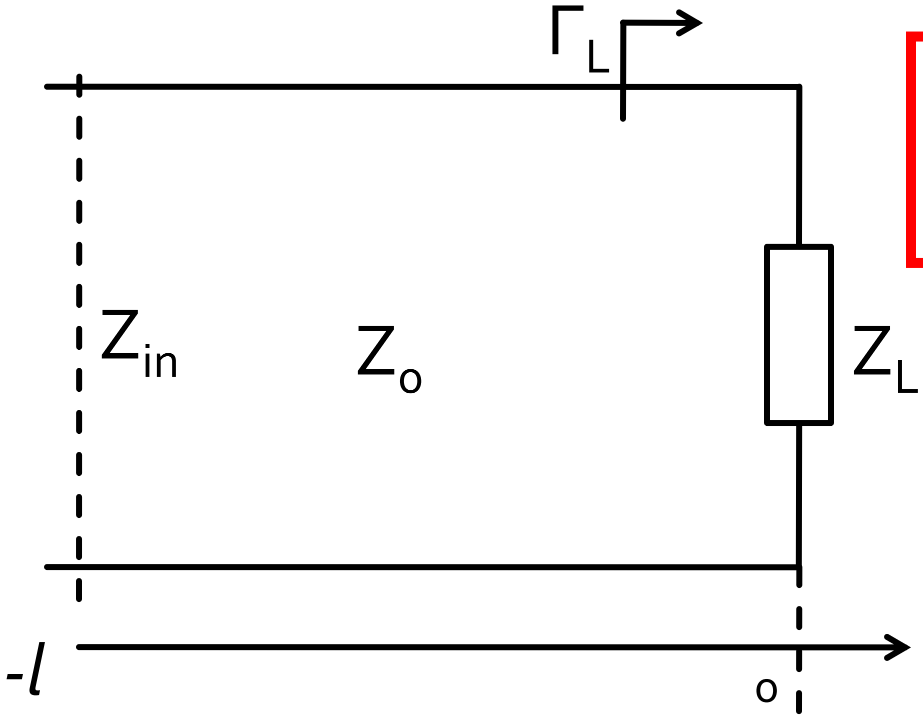


- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar !**

TEM transmission lines

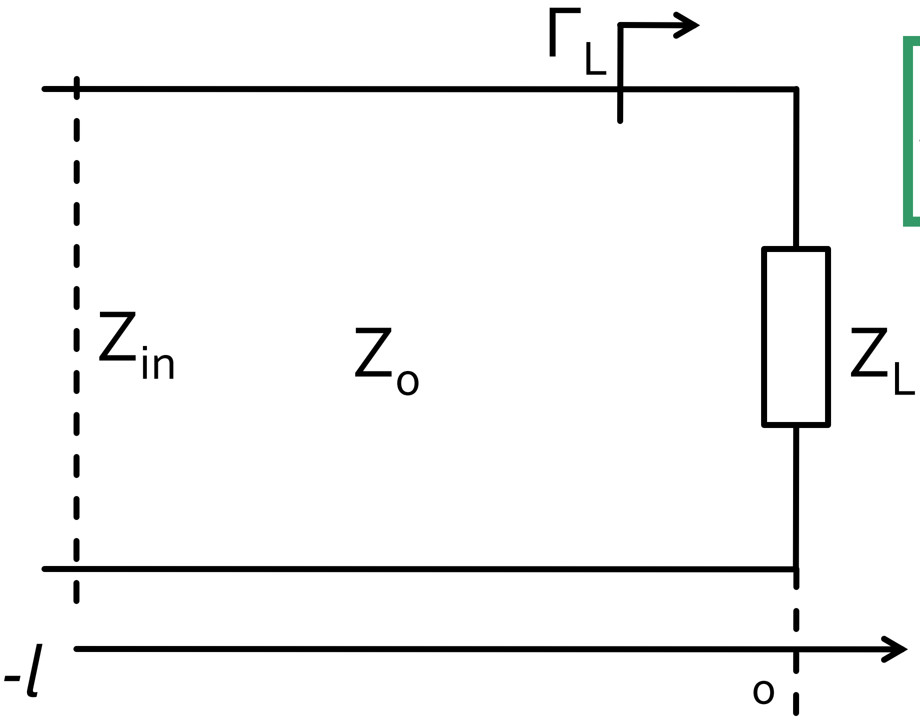
The lossless line

- input impedance of a length $\boldsymbol{l}$ of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line +/-



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z}$$

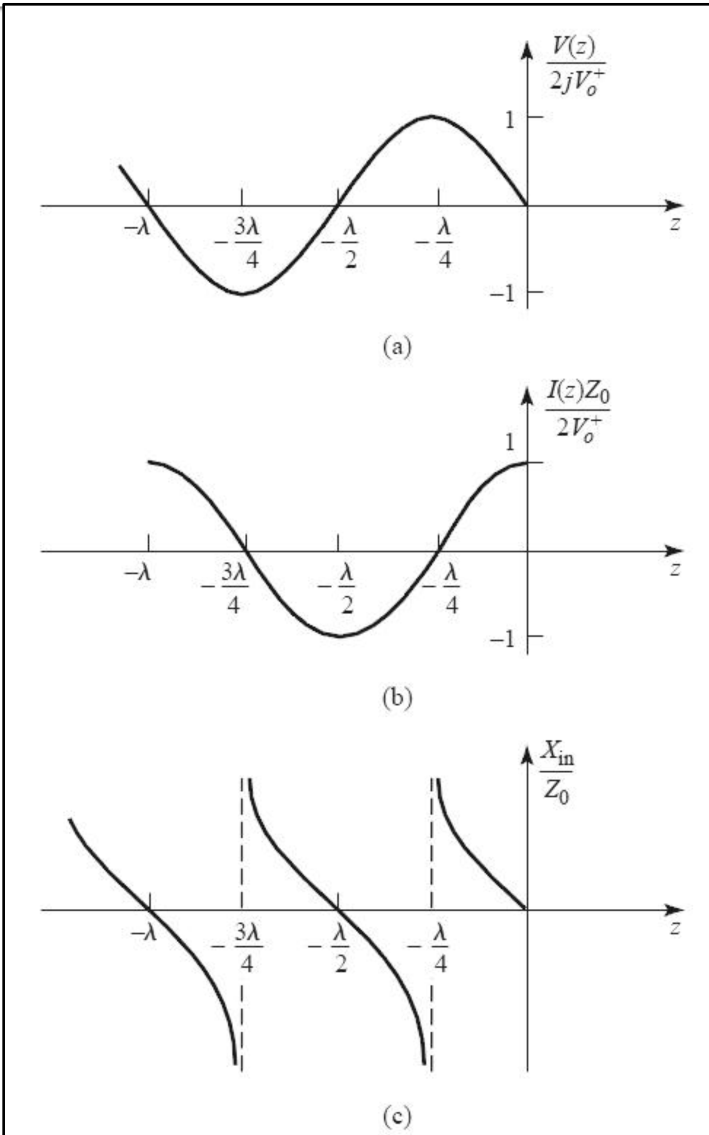
$$\Gamma(-l) = \Gamma(0) \cdot e^{-2 j \cdot \beta \cdot l}$$

Short-circuited transmission line

- purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

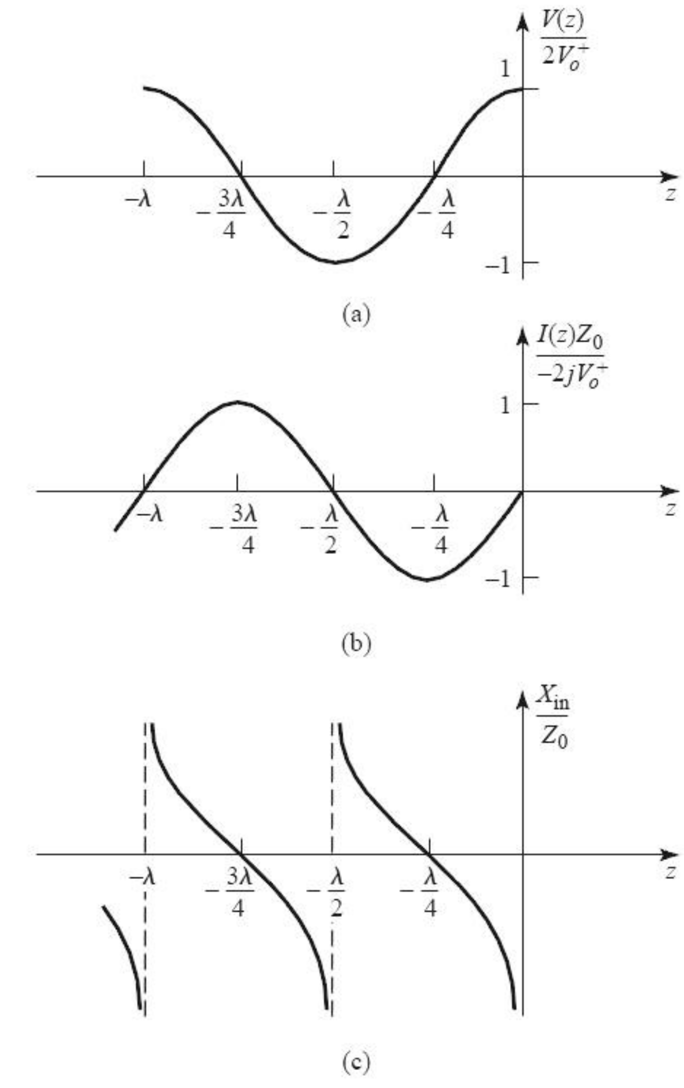


Open-circuited transmission line

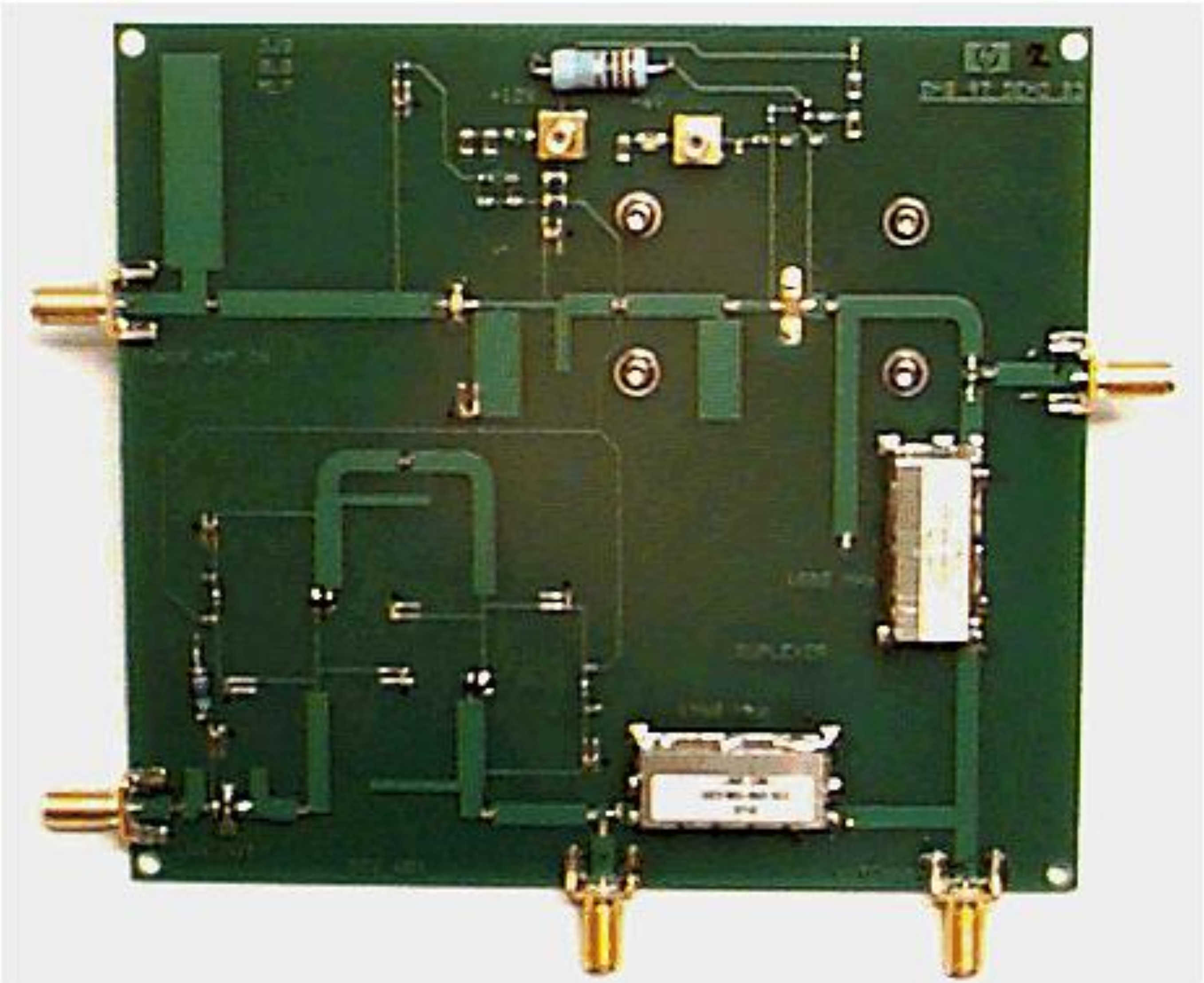
- purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

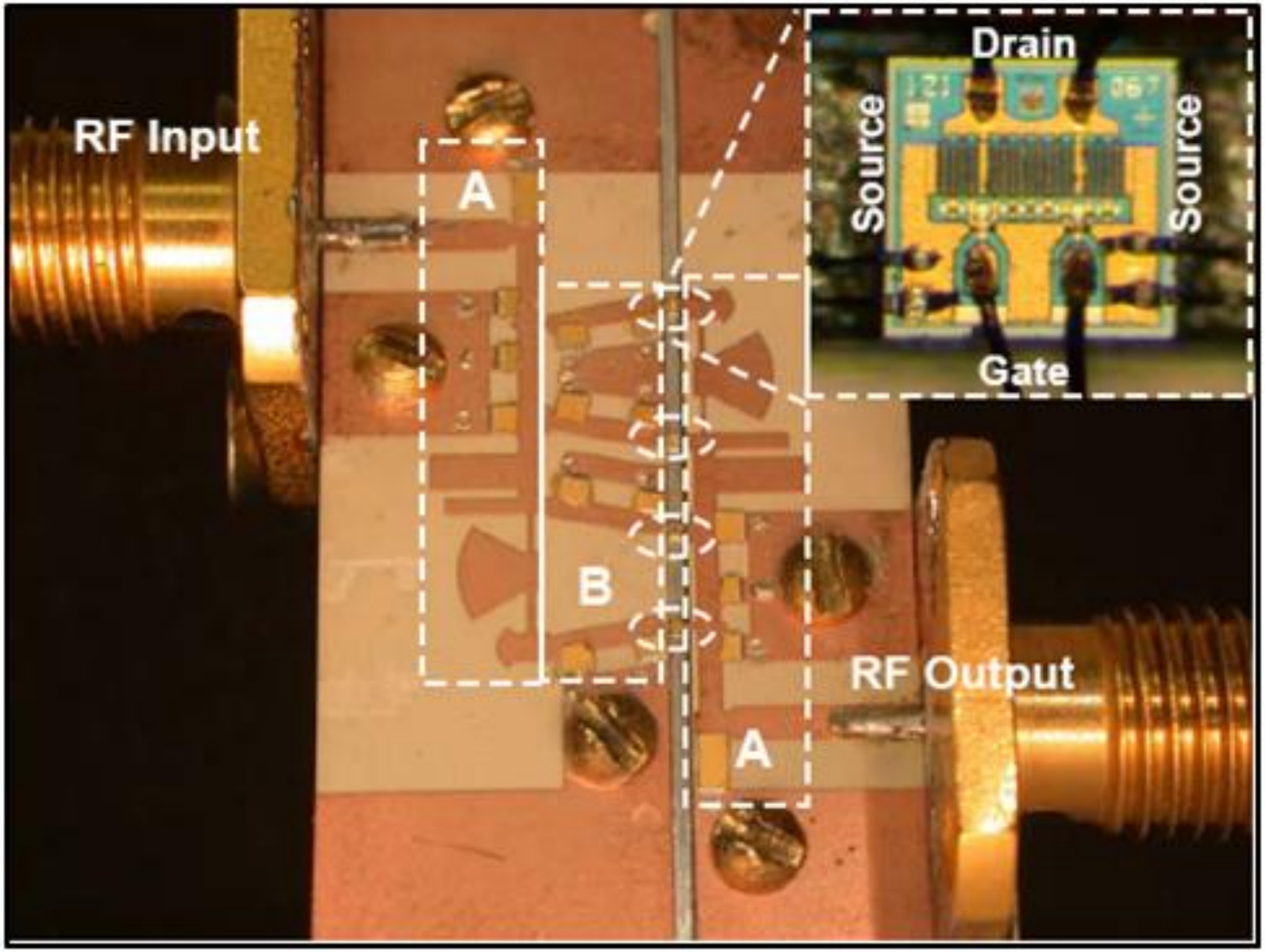
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



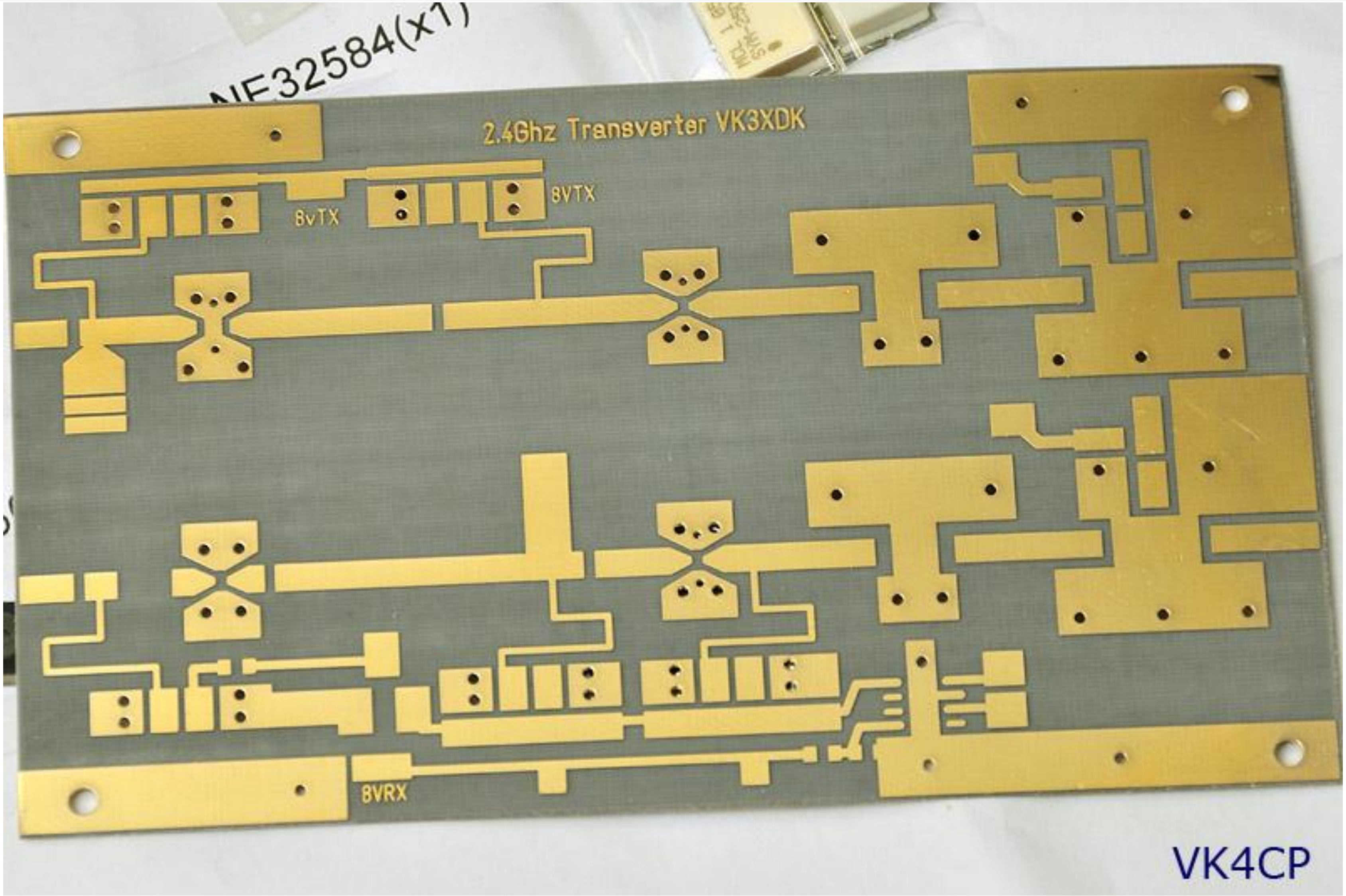
Examples



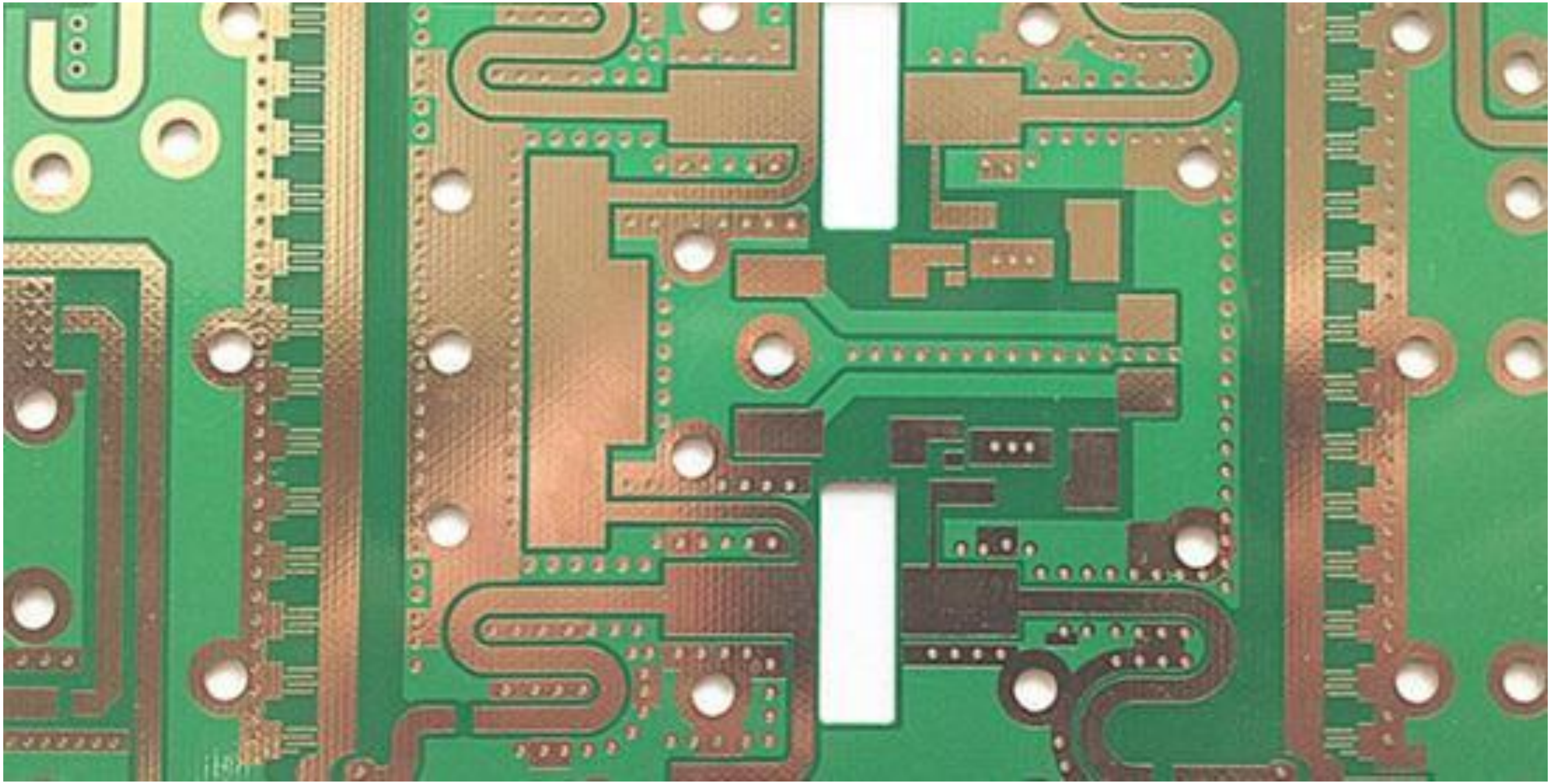
Examples



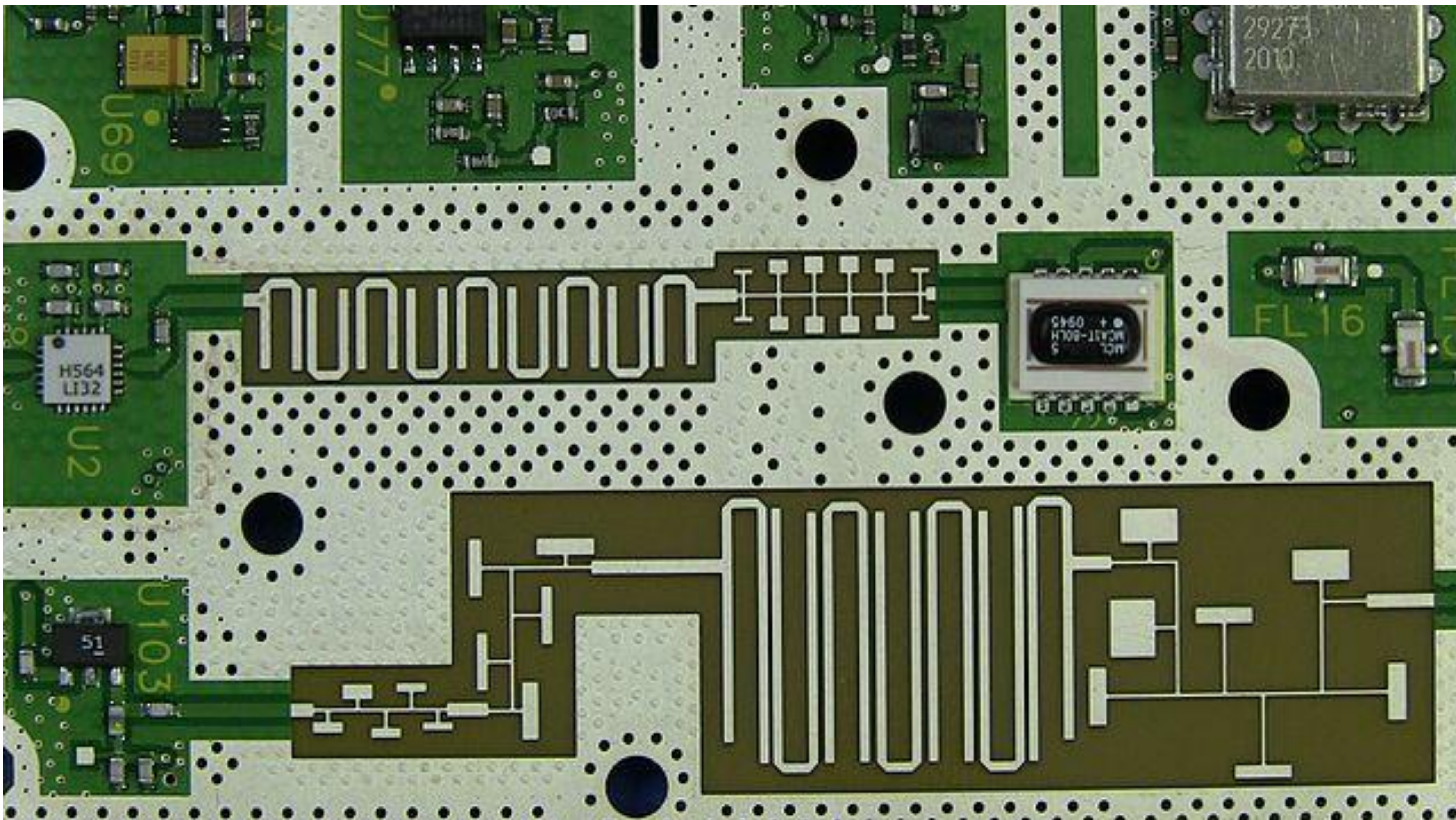
Examples



Examples



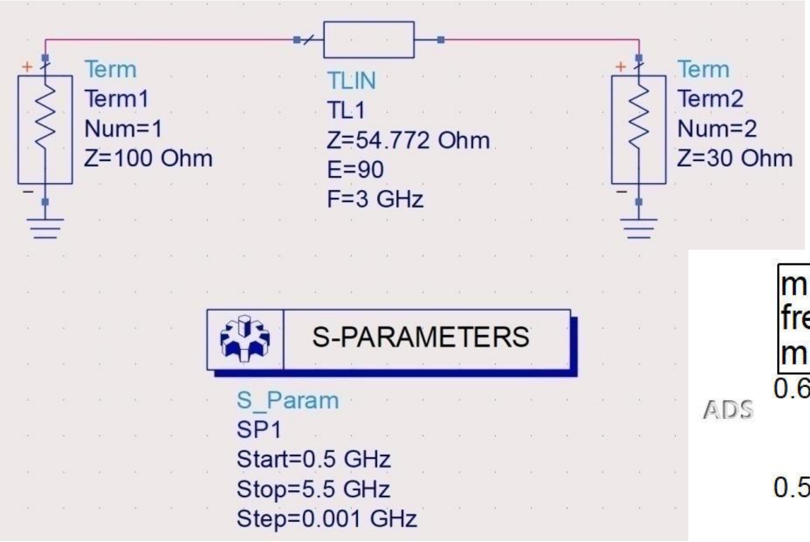
Examples

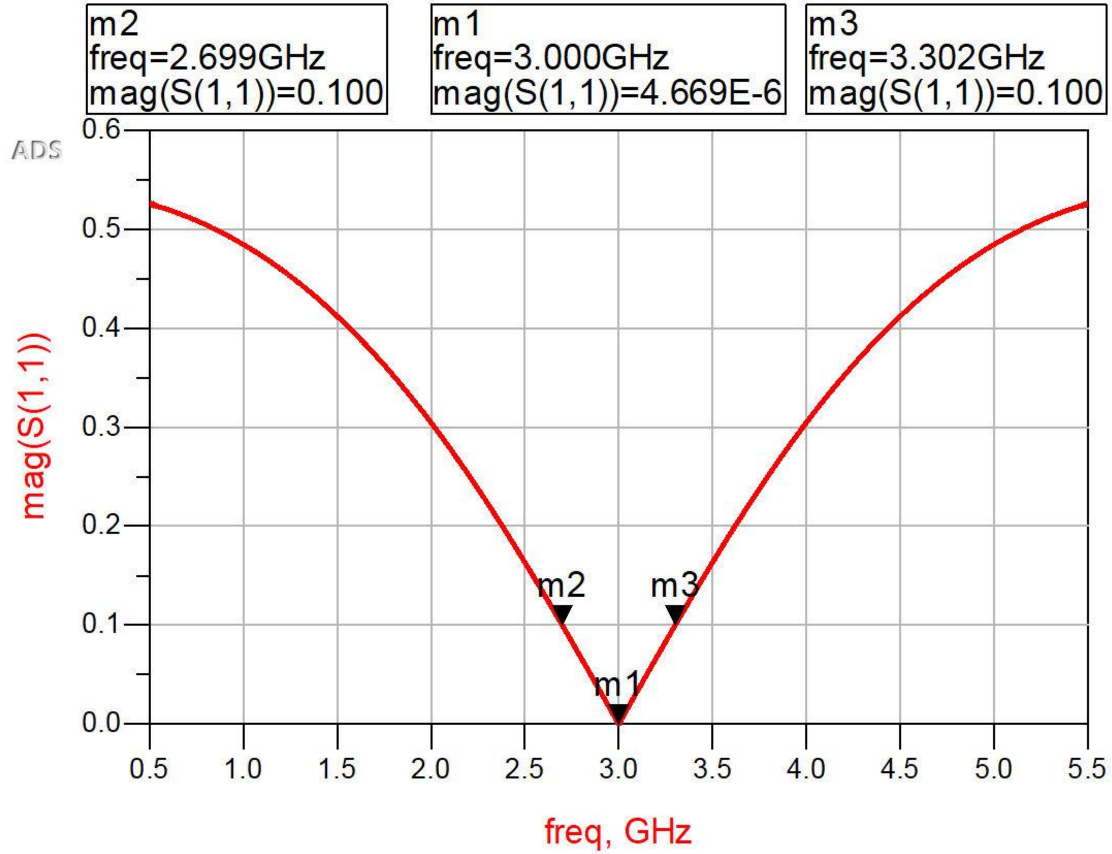


Impedance Matching with Impedance Transformers (Lab 1)

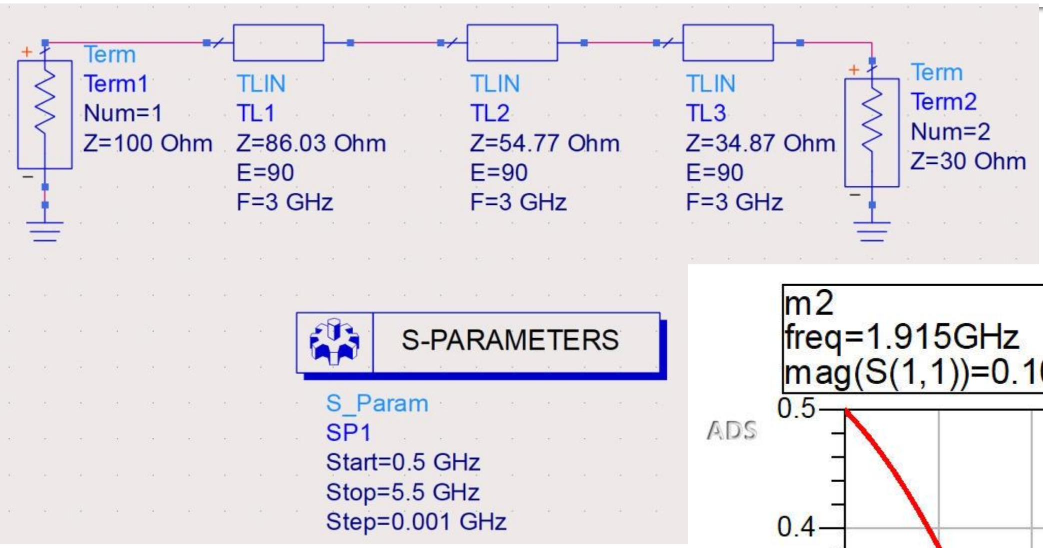
Impedance Matching

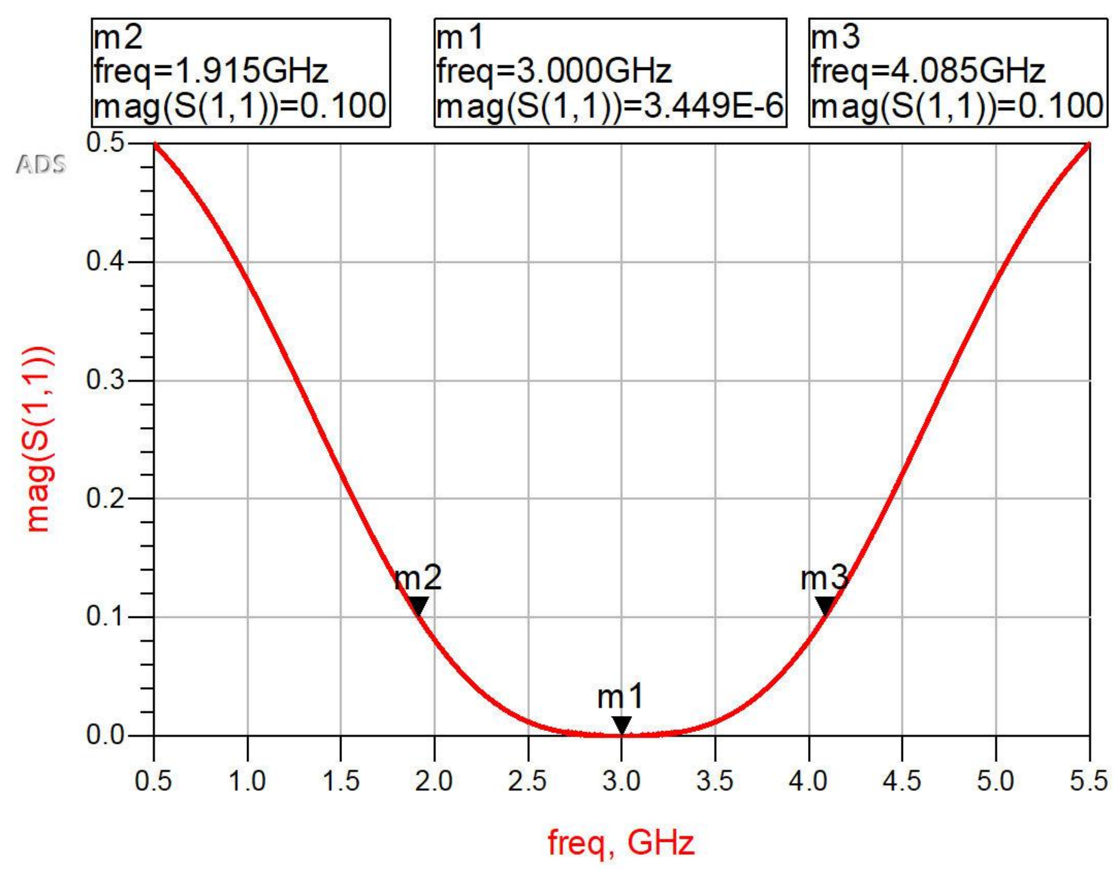
The quarter-wave transformer



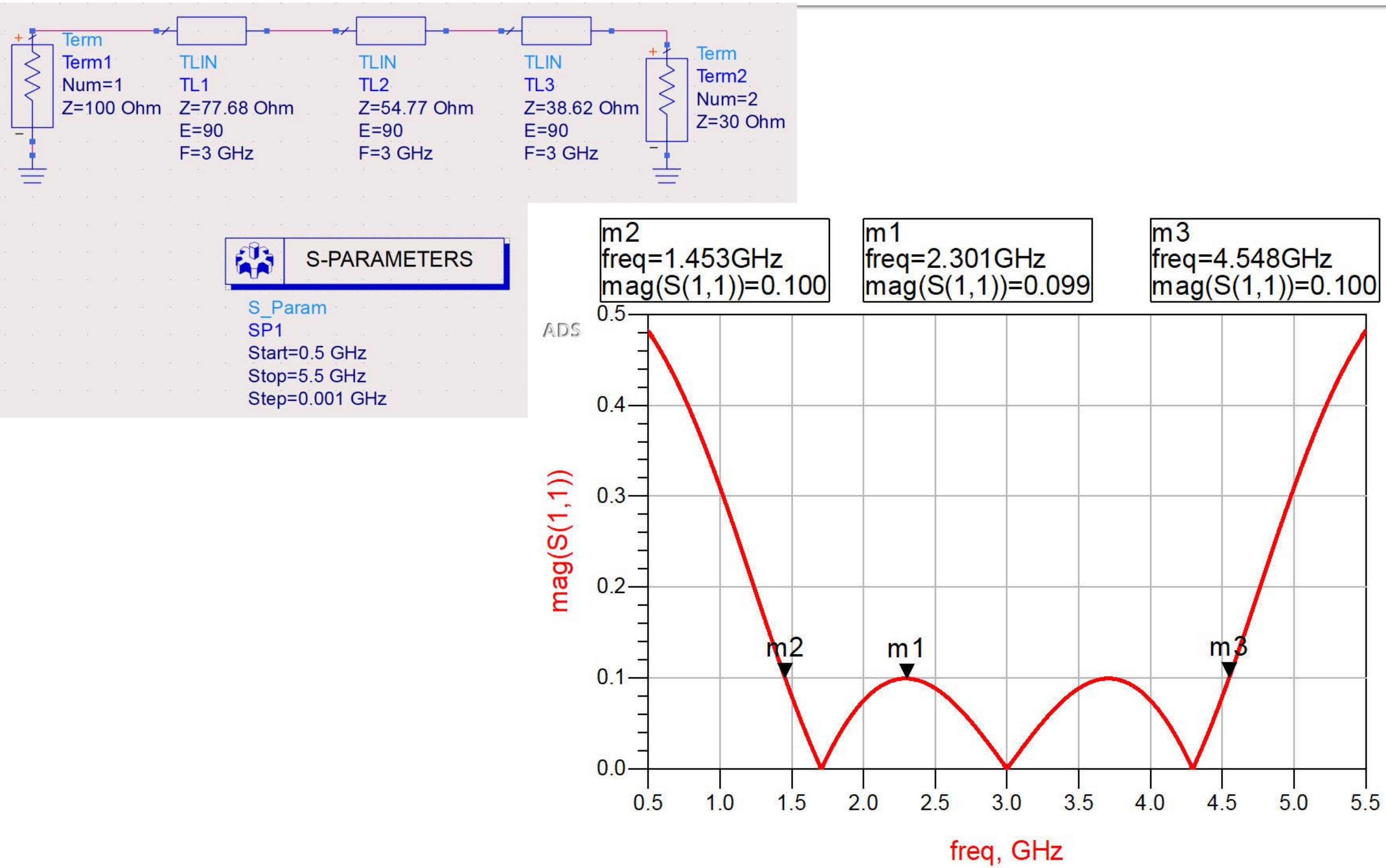


Binomial multisection transformer





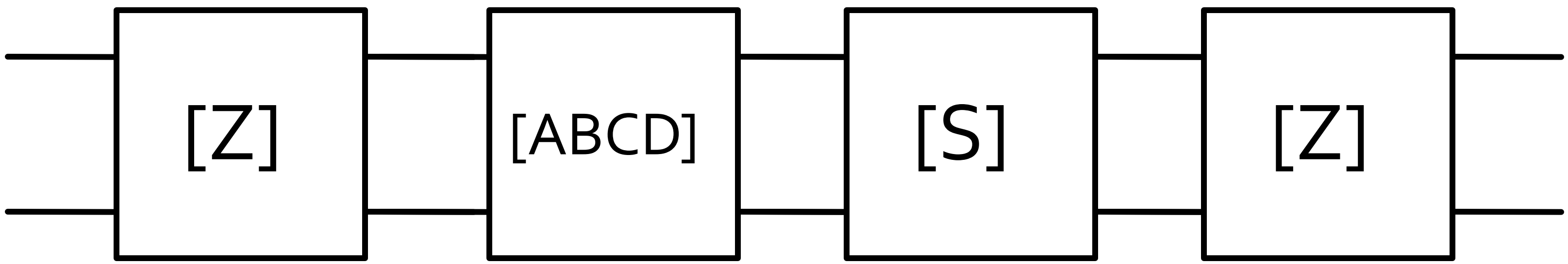
Chebyshev multisection transformer



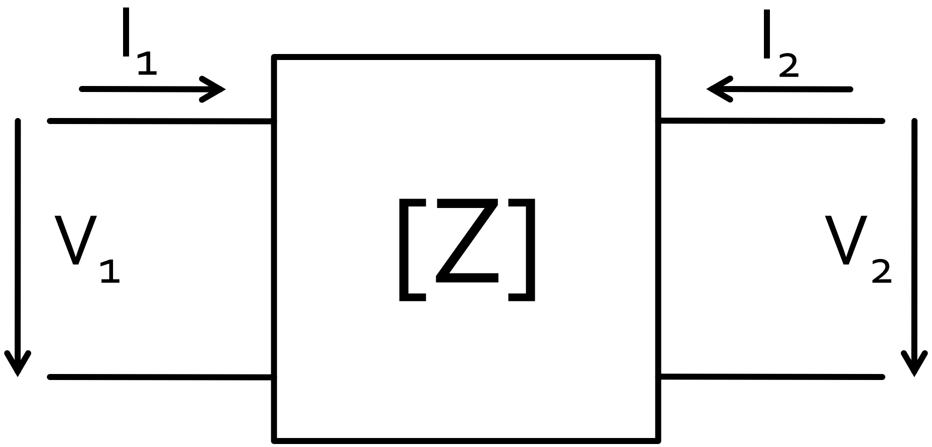
Microwave Network Analysis

Network Analysis

- We try to separate a complex circuit into individual blocks
- These are analyzed separately (decoupled from the rest of the circuit) and are characterized only by the port level signals (**black box**)
- Network-level analysis allows you to put together individual block results and get a total result for the entire circuit



Impedance matrix – Z



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

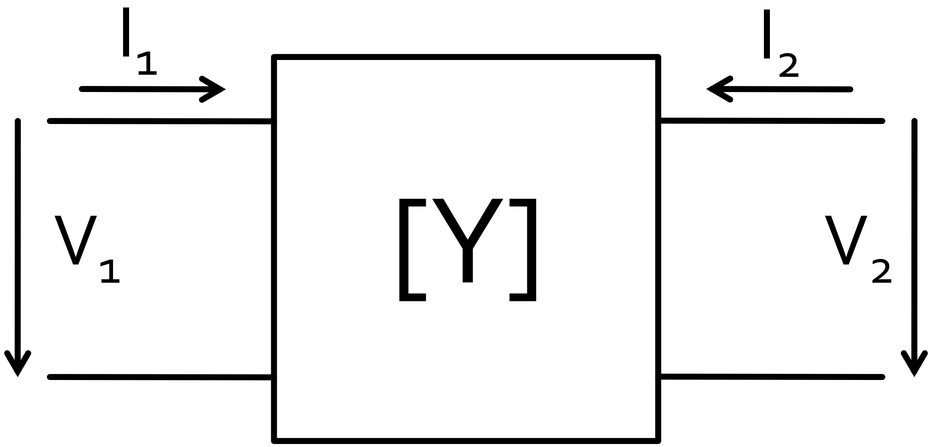
$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$
$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$$V_1 = Z_{11} \cdot I_1 \Big|_{I_2=0} \qquad Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

- Z11 – input impedance with open-circuited output

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

Admittance matrix – Y



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

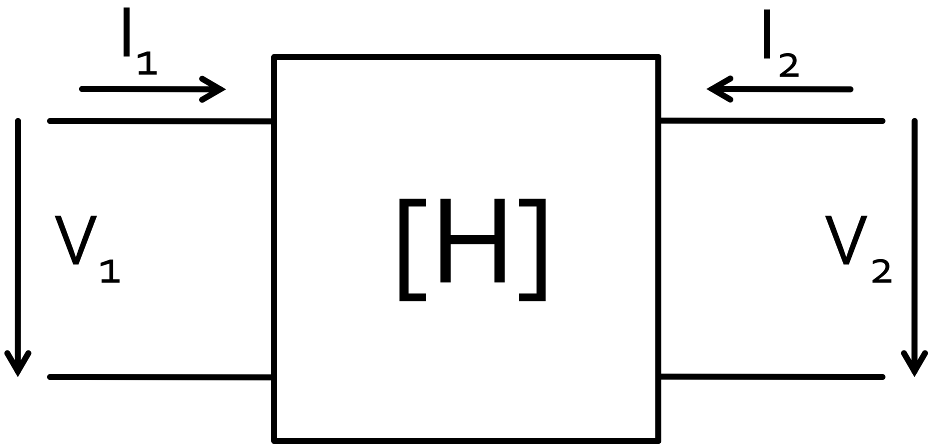
$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2$$
$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2$$

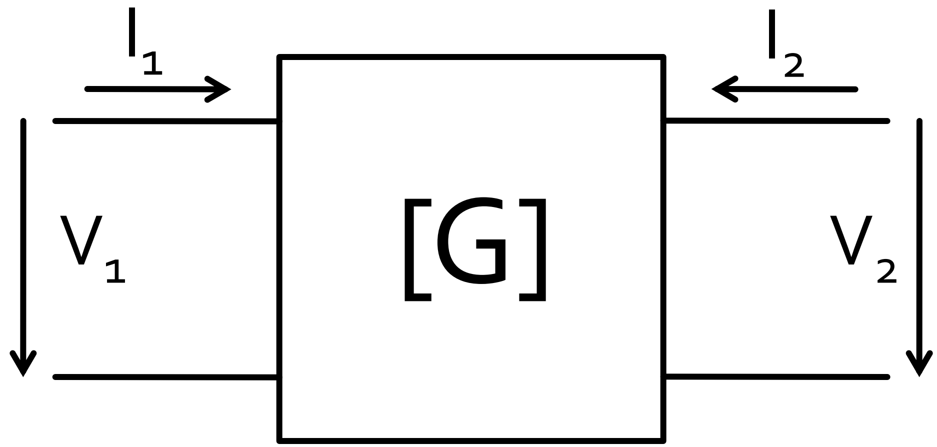
$$I_1 = Y_{11} \cdot V_1 \Big|_{V_2=0} \qquad Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

- Y11 – input admittance with short-circuited output

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \qquad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

Hybrid matrices – H and G





$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$H_{21} = \left. \frac{I_2}{I_1} \right|_{V_2 = 0 \; sau \; H_{22} \to \infty}$$

- h_{21E} widely used for Bipolar Transistors, common emitter topology (or β, h_{22E} >>)

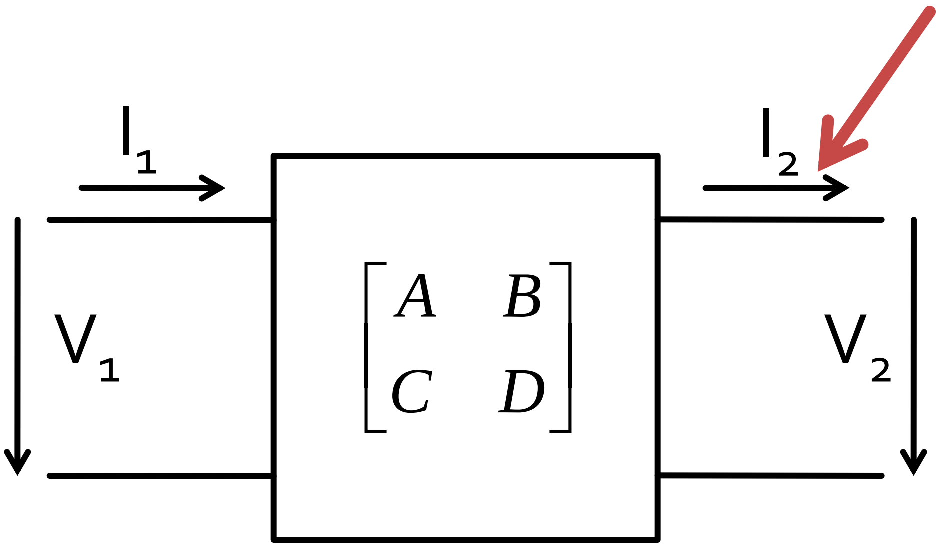
Network Analysis

- Each matrix is best suited for a particular mode of port excitation (V, I)
  - matrix H in common emitter connection for TB: I_B, V_{CE}
  - matrices provide the associated quantities depending on the "attack" ones
- Traditional notation of Z, Y, G, H parameters is in lowercase (z, y, g, h)
- In microwave analysis we prefer the notation in uppercase to avoid confusion with the normalized parameters

$$z = \frac{Z}{Z_0} \qquad y = \frac{Y}{Y_0} = \frac{1/Z}{1/Z_0} = \frac{Z_0}{Z} = Z_0 \cdot Y$$

$$z_{11} = \frac{Z_{11}}{Z_0} \qquad y_{11} = \frac{Y_{11}}{Y_0} = Z_0 \cdot Y_{11}$$

ABCD (transmission) matrix



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

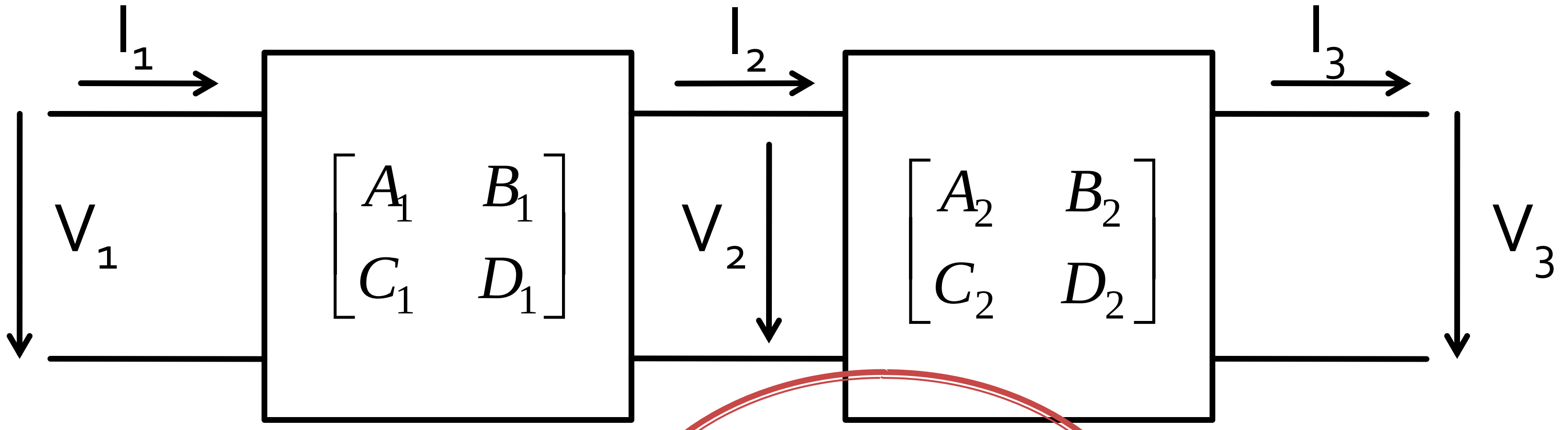
$$V_1 = A \cdot V_2 + B \cdot I_2$$
$$I_1 = C \cdot V_2 + D \cdot I_2$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{A \cdot D - B \cdot C} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

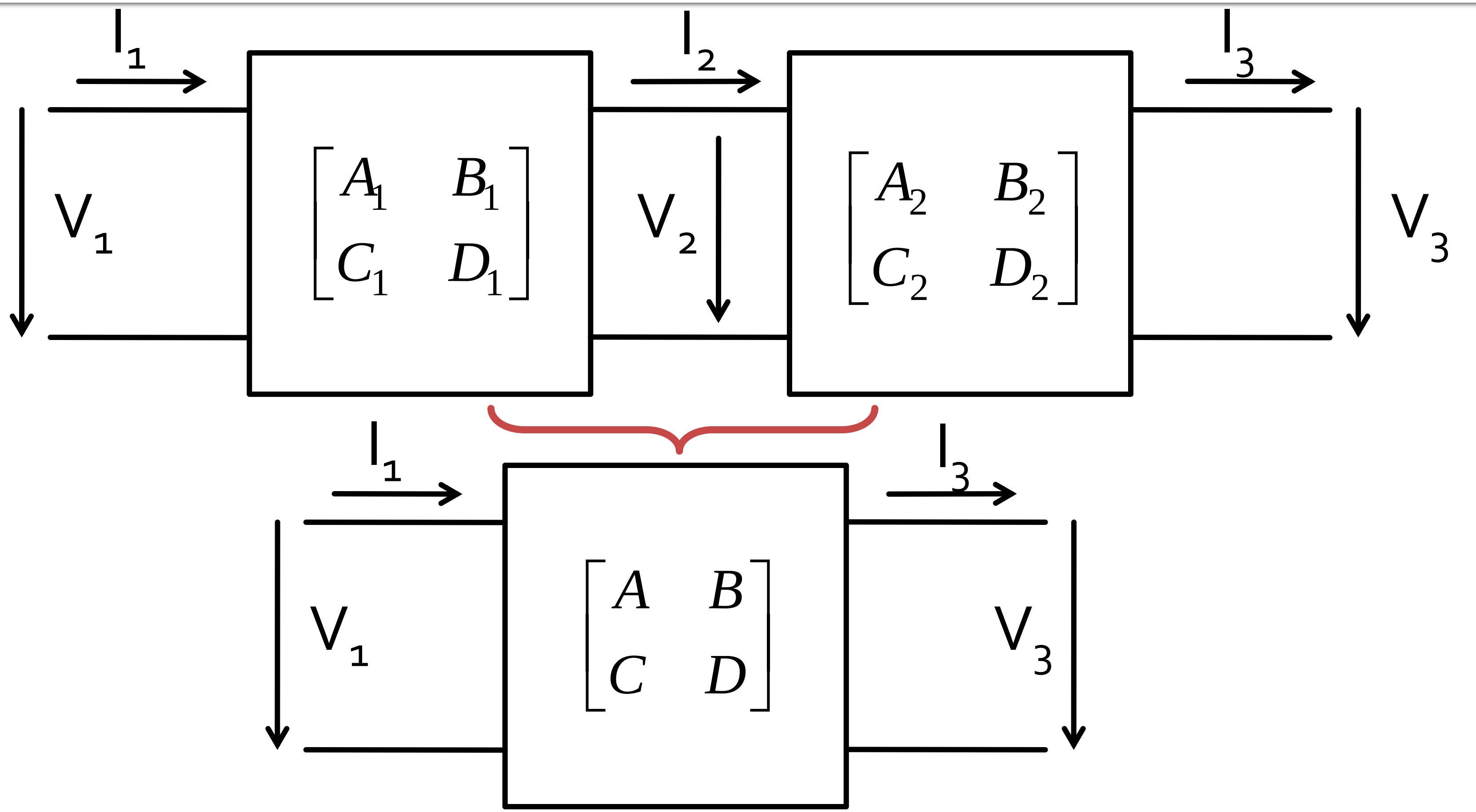
ABCD (transmission) matrix

- This 2X2 matrix characterizes the "input"/"output" relation
- Allows easy chaining of multiple two-ports



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

ABCD (transmission) matrix



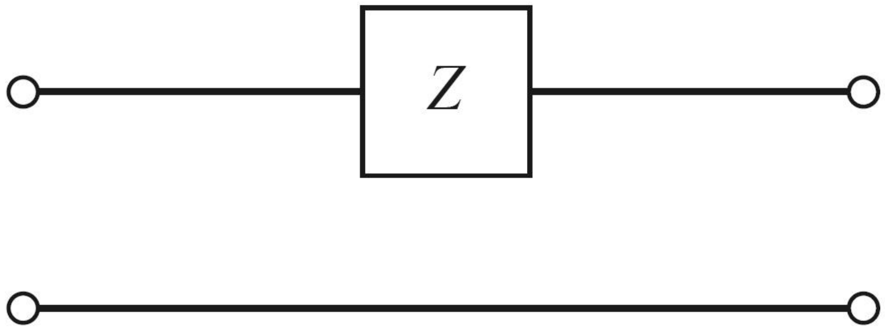
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

ABCD (transmission) matrix

- suitable **only** for two-port networks (Z, Y can be easily extended for multiport / n-ports)
- allows easy coupling of multiple elements
- allows the calculation of complex circuits with one input and one output by breaking them in individual component blocks
- a library of ABCD matrices for elementary two-port networks can be built up

Library of ABCD matrices

- Series impedance



$$A = 1 \qquad B = Z$$
$$C = 0 \qquad D = 1$$

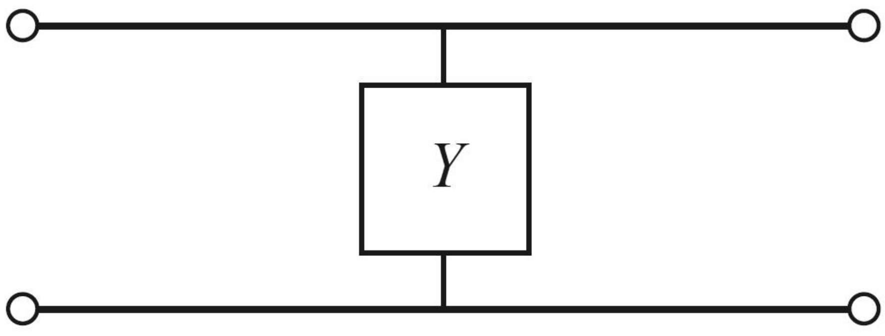
$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0 \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_1}{I_1} = 1$$

Library of ABCD matrices

- Shunt admittance



$$A = 1 \qquad B = 0$$

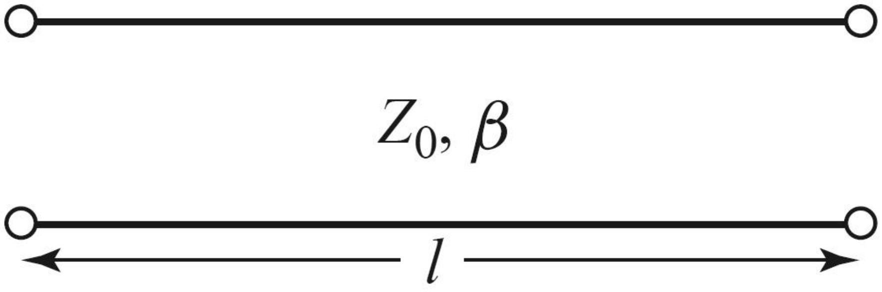
$$C = Y \qquad D = 1$$

$$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

Homework!

Library of ABCD matrices

- Transmission line



$$A = \cos \beta \cdot l$$

$$B = j \cdot Z_0 \cdot \sin \beta \cdot l$$

$$C = j \cdot Y_0 \cdot \sin \beta \cdot l$$

$$D = \cos \beta \cdot l$$

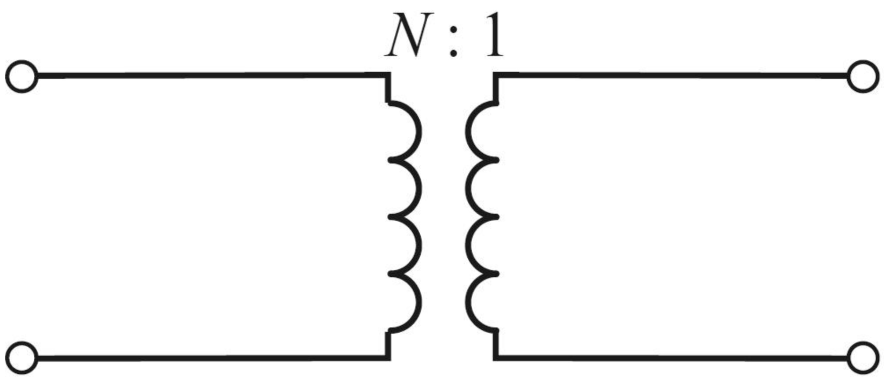
Homework!

$$\begin{bmatrix} \cos \beta \cdot l & j \cdot Z_0 \cdot \sin \beta \cdot l \\ j \cdot Y_0 \cdot \sin \beta \cdot l & \cos \beta \cdot l \end{bmatrix}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Library of ABCD matrices

- Transformer



$$A = N \qquad B = 0$$

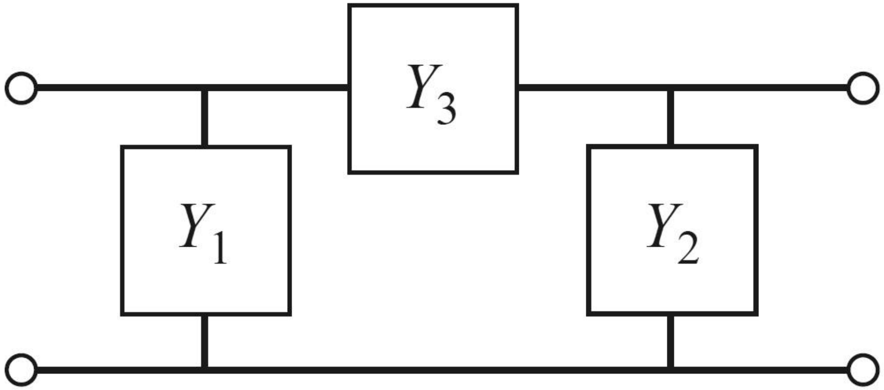
$$C = 0 \qquad D = \frac{1}{N}$$

$$\begin{bmatrix} N & 0 \\ 0 & \frac{1}{N} \end{bmatrix}$$

Homework!

Library of ABCD matrices

- π network



$$A = 1 + \frac{Y_2}{Y_3}$$

$$B = \frac{1}{Y_3}$$

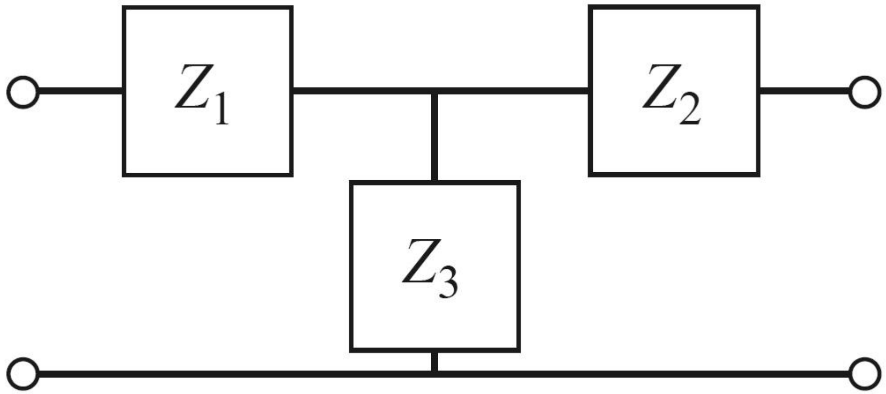
$$C = Y_1 + Y_2 + \frac{Y_1 \cdot Y_2}{Y_3}$$

$$D = 1 + \frac{Y_1}{Y_3}$$

Homework!

Library of ABCD matrices

- T network



$$A = 1 + \frac{Z_1}{Z_3}$$

$$B = Z_1 + Z_2 + \frac{Z_1 \cdot Z_2}{Z_3}$$

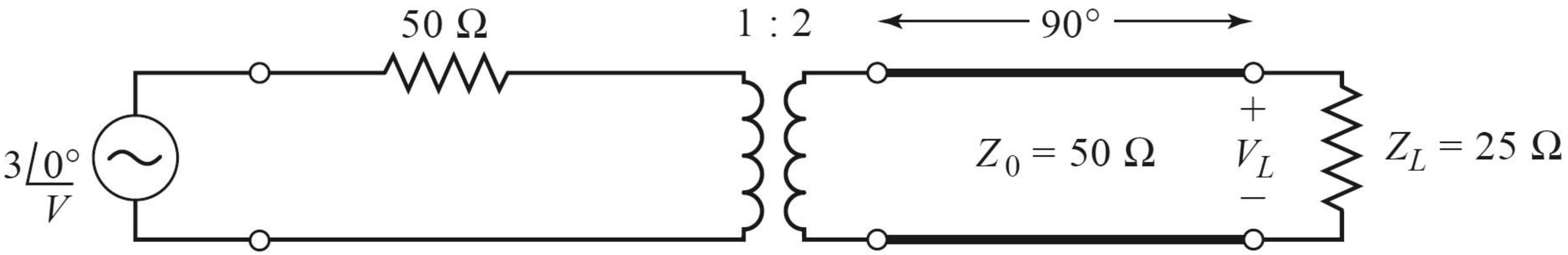
$$C = \frac{1}{Z_3}$$

$$D = 1 + \frac{Z_2}{Z_3}$$

Homework!

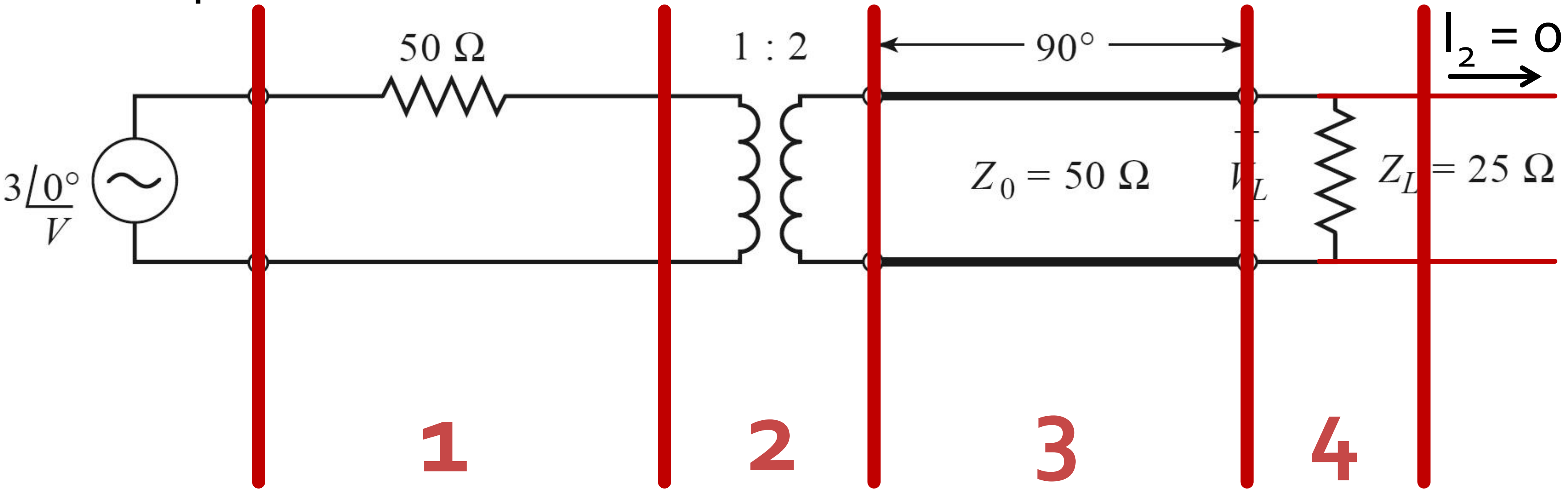
Example for ABCD matrix

- Find the voltage V_L across the load resistor in the circuit shown below



Example for ABCD matrix

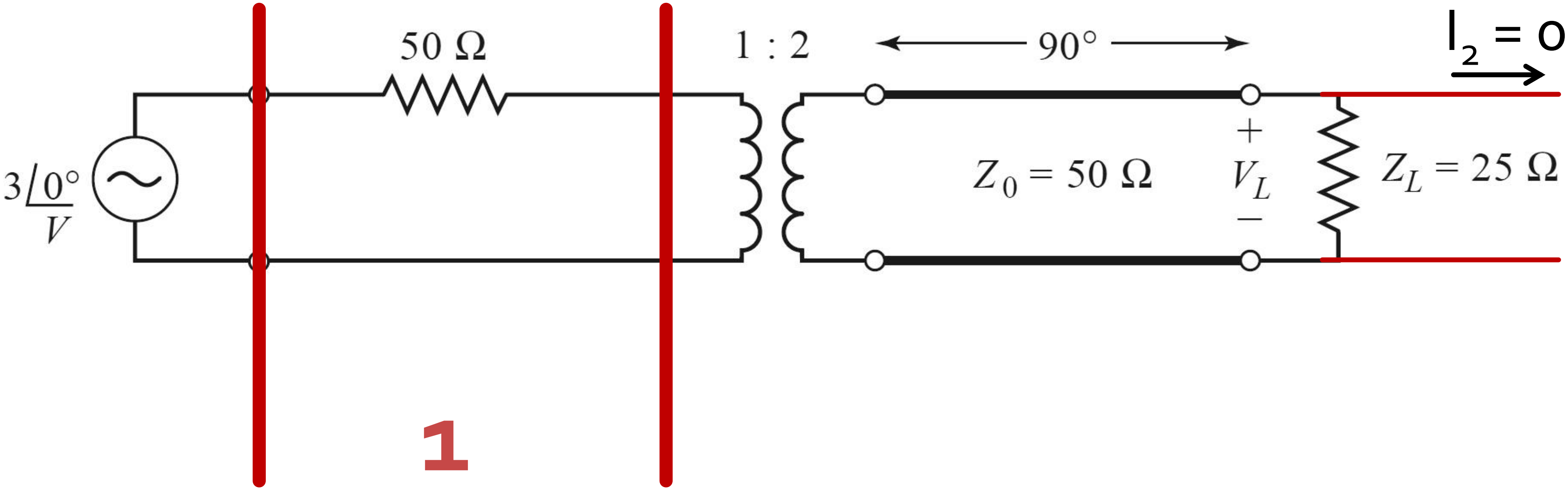
- We break the circuit in elementary sections
- Sources are left outside
- If necessary, input and output ports are created (and left open-circuited)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = M_1 \cdot M_2 \cdot M_3 \cdot M_4 \quad V_1 = A \cdot V_2 + B \cdot I_2 \Big|_{I_2=0} \quad V = A \cdot V_L \rightarrow V_L = \frac{V}{A}$$

Example for ABCD matrix

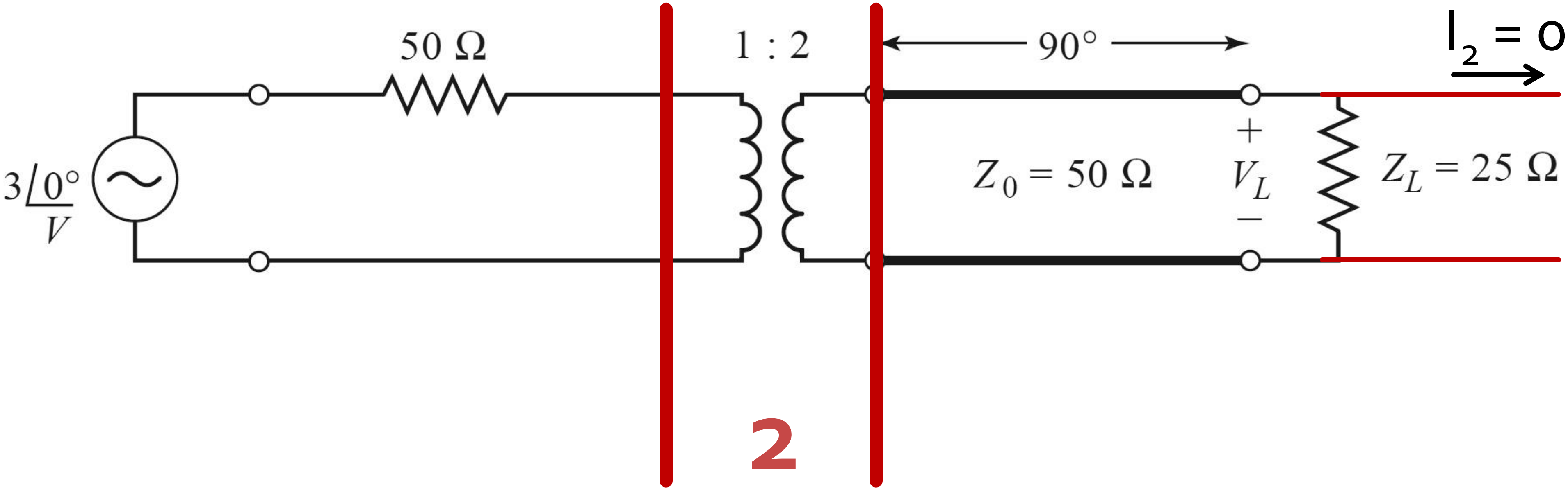
- M_1 , series impedance



$$M_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$$

Example for ABCD matrix

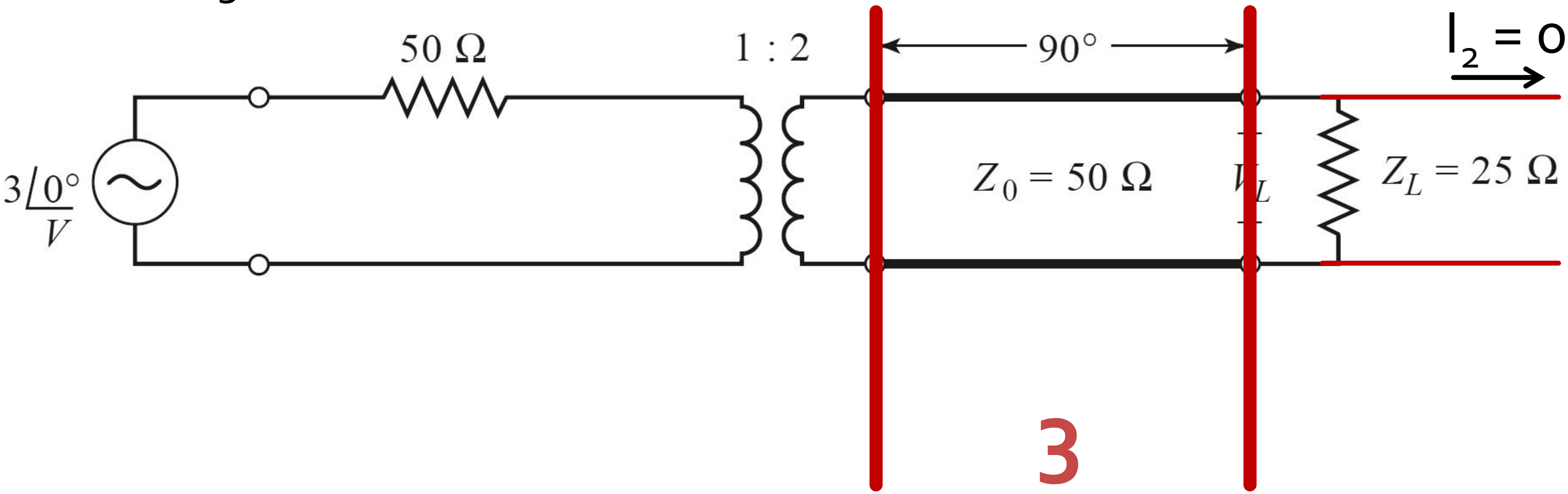
- M_2 , 1:2 transformer



$$M_2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$

Example for ABCD matrix

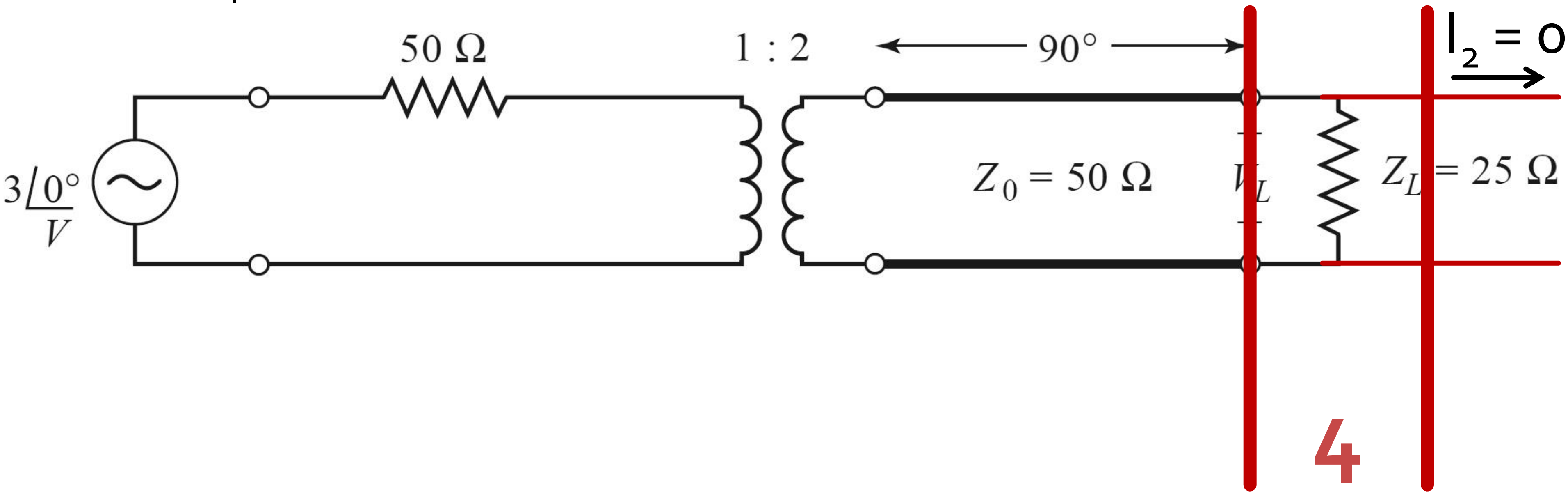
- M_3 , series transmission line, E = 90°



$$M_3 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & 50 \cdot j \\ \frac{j}{50} & 0 \end{bmatrix}$$

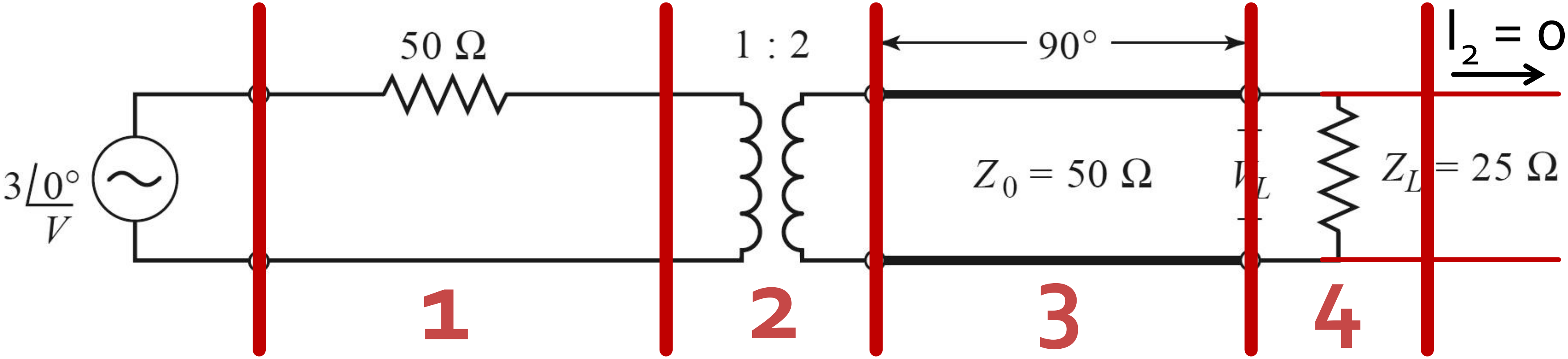
Example for ABCD matrix

- M_4 , shunt impedance/admittance



$$M_4 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{25} & 1 \end{bmatrix}$$

Example for ABCD matrix

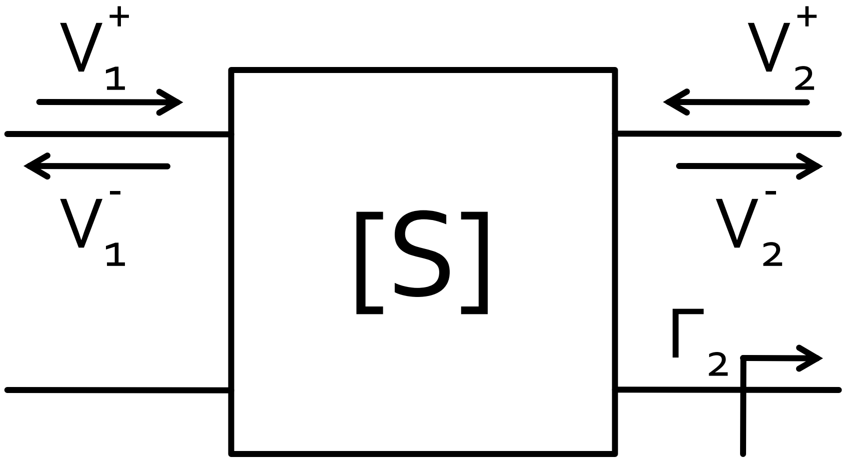


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 50 \cdot j \\ \frac{j}{50} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{1}{25} & 1 \end{bmatrix} = \begin{bmatrix} 3 \cdot j & 25 \cdot j \\ \frac{j}{25} & 0 \end{bmatrix}$$

$$V_L = \frac{V}{A} = \frac{3\angle 0°}{3 \cdot j} = 1\angle 90°$$

Scattering matrix – S

- Scattering parameters



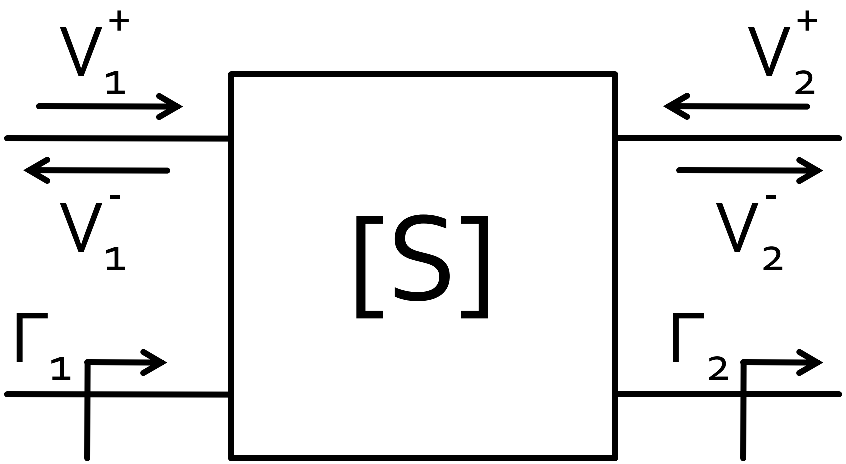
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \Gamma_1\big|_{\Gamma_2=0}$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = T_{21}\big|_{\Gamma_2=0}$$

- S11 is the reflection coefficient seen looking into port ***1*** when port ***2*** is terminated in matched load
- S21 is the transmission coefficient from port ***1*** (**second** index) to port ***2*** (**first** index) when port ***2*** is terminated in matched load

Scattering matrix – S

- S matrix can be extended to multiple ports

$$S_{ii} = \left.\frac{V_i^-}{V_i^+}\right|_{V_k^+=0,\forall k\neq i} \qquad S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0,\forall k\neq j}$$

  - S_{ii} is the reflection coefficient seen looking into port ***i*** when all other ports are terminated in matched loads
  - S_{ij} is the transmission coefficient from port ***j*** (**second** index) to port ***i*** (**first** index) when all other ports are terminated in matched loads

Properties of S matrix

- If port i is connected to a transmission line with charateristic impedance Z_{oi}

$$[Z_0] = \begin{bmatrix} Z_{01} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Z_{0n} \end{bmatrix}$$

- Lecture 3

$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z} \qquad I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

In the port's reference plane, z=o

$$V_i = V_i^+ + V_i^- \qquad I_i = \frac{V_i^+}{Z_{0i}} - \frac{V_i^-}{Z_{0i}}$$

- Relation to Z matrix

$$[Z]\cdot[I] = [V]$$

$$[Z]\cdot[I] = [Z_0]^{-1}\cdot[Z]\cdot[V^+] - [Z_0]^{-1}\cdot[Z]\cdot[V^-] \qquad [V] = [V^+] + [V^-]$$

$$[Z_0]^{-1}\cdot[Z]\cdot[V^+] - [Z_0]^{-1}\cdot[Z]\cdot[V^-] = [V^+] + [V^-] \qquad ([Z]-[Z_0])\cdot[V^+] = ([Z]+[Z_0])\cdot[V^-]$$

$$[V^-] = [S]\cdot[V^+] \qquad \boxed{[S] = ([Z]-[Z_0])\cdot([Z]+[Z_0])^{-1}}$$

A Shift in Reference Planes

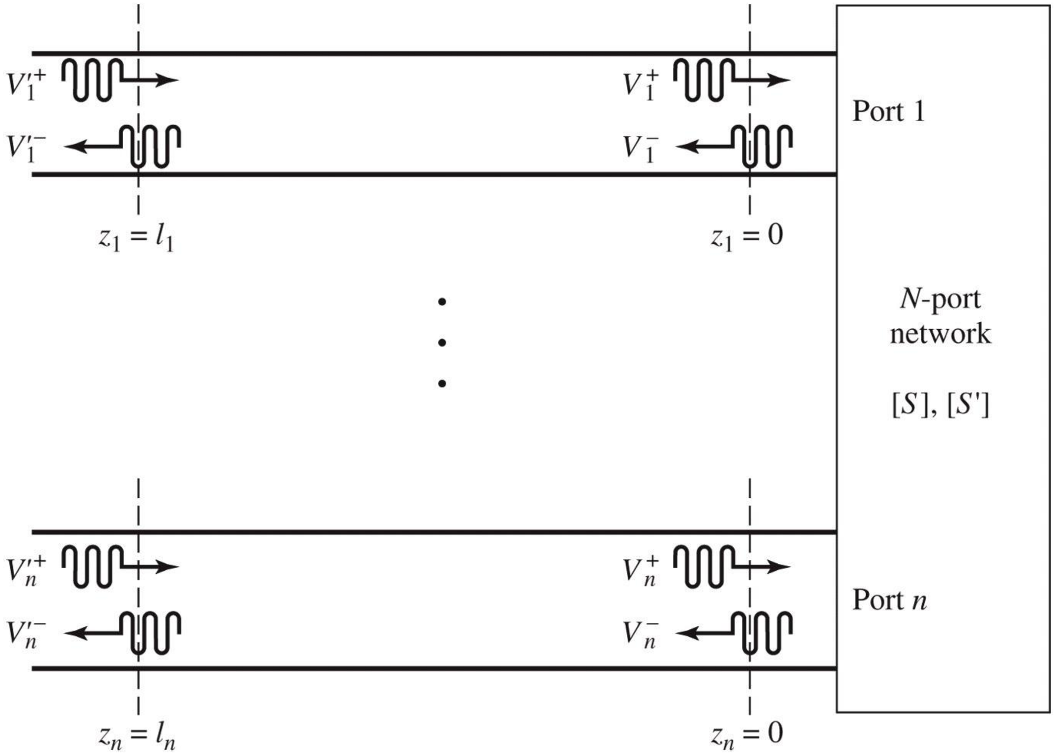


Figure 4.9

$$[S'] = \begin{bmatrix} e^{-j\cdot\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j\cdot\theta_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & e^{-j\cdot\theta_N} \end{bmatrix} \cdot [S] \cdot \begin{bmatrix} e^{-j\cdot\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j\cdot\theta_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & e^{-j\cdot\theta_N} \end{bmatrix}$$

Properties of S matrix

- Reciprocal networks (no active circuits, no ferrites)

$$Z_{ij} = Z_{ji}, \forall j \neq i$$

$$Y_{ij} = Y_{ji}, \forall j \neq i$$

$$S_{ij} = S_{ji}, \forall j \neq i$$

$$[S] = [S]^t$$

- Lossless networks

$$\mathrm{Re}\{Z_{ij}\} = 0, \forall i, j$$

$$\mathrm{Re}\{Y_{ij}\} = 0, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{kj}^{*} = \delta_{ij}, \forall i, j$$

$$[S]^{*} \cdot [S]^{t} = [1]$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^{*} = 1$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{kj}^{*} = 0, \forall i \neq j$$

Generalized Scattering Parameters

- The total voltage and current on a transmission line in terms of the incident and reflected voltage wave amplitudes

$$V = V_0^+ + V_0^- \qquad I = \frac{1}{Z_0}\cdot\left(V_0^+ - V_0^-\right)$$

In the port's reference plane, z=o

- We find the incident and reflected voltage wave amplitudes

$$V_0^+ = \frac{V + Z_0 \cdot I}{2} \qquad V_0^- = \frac{V - Z_0 \cdot I}{2}$$

- The average power delivered to a load :

$$P_L = \frac{1}{2}\mathrm{Re}\left\{V I^*\right\} = \frac{1}{2Z_0}\mathrm{Re}\left\{\left|V_0^+\right|^2 - V_0^+ V_0^{-*} + V_0^{+*} V_0^- - \left|V_0^-\right|^2\right\}$$

$$= \frac{1}{2Z_0}\left(\left|V_0^+\right|^2 - \left|V_0^-\right|^2\right),$$

Generalized Scattering Parameters

- We define the power wave amplitudes a and b

$$a = \frac{V + Z_R \cdot I}{2 \cdot \sqrt{R_R}}$$ the incident power wave

$$Z_R = R_R + j \cdot X_R$$

Any complex impedance, named reference impedance

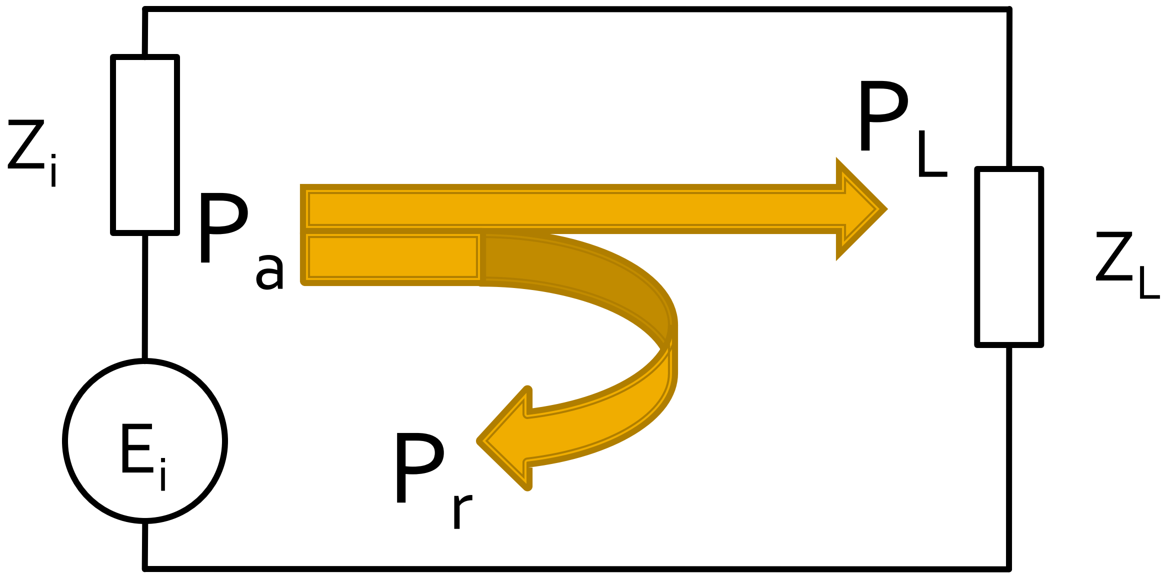
$$b = \frac{V - Z_R^* \cdot I}{2 \cdot \sqrt{R_R}}$$ the reflected power wave

- Total voltage and current in terms of the power wave amplitudes

$$V = \frac{Z_R^* \cdot a + Z_R \cdot b}{\sqrt{R_R}}$$

$$I = \frac{a - b}{\sqrt{R_R}}$$

Reflection and power / Model – L2



$$P_a = \frac{|E_i|^2}{4R_i}$$

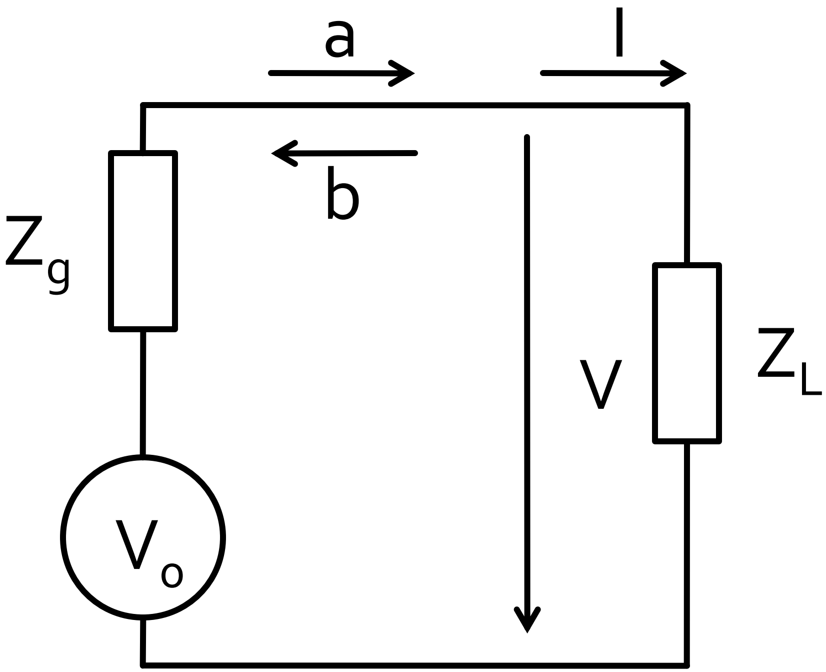
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$P_r = \frac{|E_i|^2}{4R_i} \cdot \left[\frac{(R_i - R_L)^2 + (X_i + X_L)^2}{(R_i + R_L)^2 + (X_i + X_L)^2} \right] = P_a \cdot |\Gamma|^2$$

- Γ, power reflection coefficient

Power waves



$$P_L = \frac{1}{2}\cdot \mathrm{Re}\left\{V \cdot I^*\right\}$$

$$P_L = \frac{1}{2}\cdot \mathrm{Re}\left\{\frac{Z_R^* \cdot a + Z_R \cdot b}{\sqrt{R_R}} \cdot \left(\frac{a-b}{\sqrt{R_R}}\right)^*\right\}$$

$$P_L = \frac{1}{2R_R}\cdot \mathrm{Re}\left\{Z_R^* \cdot |a|^2 - Z_R^* \cdot a \cdot b^* + Z_R \cdot a^* \cdot b - Z_R \cdot |b|^2\right\}$$

$$P_L = \frac{1}{2}\cdot |a|^2 - \frac{1}{2}\cdot |b|^2$$

$$\Gamma_p = \frac{b}{a} = \frac{V - Z_R^* \cdot I}{V + Z_R \cdot I} = \frac{Z_L - Z_R^*}{Z_L + Z_R}$$

Power waves

$$V = \frac{V_0 \cdot Z_L}{Z_g + Z_L} \qquad I = \frac{V_0}{Z_g + Z_L} \qquad P_L = \frac{V_0^2}{2} \cdot \frac{R_L}{\left|Z_g + Z_L\right|^2}$$

- If we choose $Z_R = Z_L^*$

$$a = \frac{V + Z_R \cdot I}{2 \cdot \sqrt{R_R}} = V_0 \cdot \frac{\frac{Z_L}{Z_g + Z_L} + \frac{Z_L^*}{Z_g + Z_L}}{2 \cdot \sqrt{R_L}} = V_0 \cdot \frac{\sqrt{R_L}}{Z_g + Z_L}$$

$$b = \frac{V - Z_R^* \cdot I}{2 \cdot \sqrt{R_R}} = V_0 \cdot \frac{\frac{Z_L}{Z_g + Z_L} - \frac{Z_L}{Z_g + Z_L}}{2 \cdot \sqrt{R_L}} = 0$$

$$P_L = \frac{1}{2} \cdot |a|^2 = \frac{V_0^2}{2} \cdot \frac{R_L}{\left|Z_g + Z_L\right|^2}$$

Power waves

- When the load is conjugately matched to the generator

$$Z_g = Z_L^* \qquad P_{L\max} = \frac{1}{2}\cdot|a|^2 = \frac{V_0^2}{8\cdot R_L}$$

- Power reflection: L2

$$Z_L = Z_i^* \qquad P_{L\max} \equiv P_a \qquad \Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$

$$Z_L \neq Z_i^* \qquad P_r = P_a \cdot |\Gamma|^2 \qquad P_L = P_a - P_r = P_a - P_a\cdot|\Gamma|^2 = P_a\cdot\left(1-|\Gamma|^2\right)$$

- Power reflection: L4

$$P_{L\max} \equiv P_a = \frac{1}{2}\cdot|a|^2 \qquad P_L = \frac{1}{2}\cdot|a|^2 - \frac{1}{2}\cdot|b|^2 \qquad \Gamma_p = \frac{b}{a} = \frac{V - Z_R^*\cdot I}{V + Z_R\cdot I} = \frac{Z_L - Z_R^*}{Z_L + Z_R}$$

$$P_L = \frac{1}{2}\cdot|a|^2 - \frac{1}{2}\cdot|a|^2\cdot|\Gamma_p|^2 \qquad P_L = P_a\cdot\left(1-|\Gamma_p|^2\right) \qquad P_r = P_a\cdot|\Gamma_p|^2 = \frac{1}{2}\cdot|b|^2$$

Power waves

- To define the scattering matrix for power waves for an N-port network

$$[Z_R] = \begin{bmatrix} Z_{R1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Z_{Rn} \end{bmatrix} \qquad [F] = \begin{bmatrix} 1/2\sqrt{R_{R1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/2\sqrt{R_{Rn}} \end{bmatrix}$$

$$[a] = [F] \cdot ([V] + [Z_R] \cdot [I])$$

$$[b] = [F] \cdot ([V] - [Z_R]^* \cdot [I])$$

$$[z] \cdot [I] = [V]$$

Power waves for N ports

$$[b]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}\cdot[a]$$

- The scattering matrix for power waves, $[S_p]$

$$[b]=\left[S_p\right]\cdot[a]$$

$$\left[S_p\right]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}$$

- But: $[S]=\left([Z]-[Z_0]\right)\cdot\left([Z]+[Z_0]\right)^{-1}$

- Typically

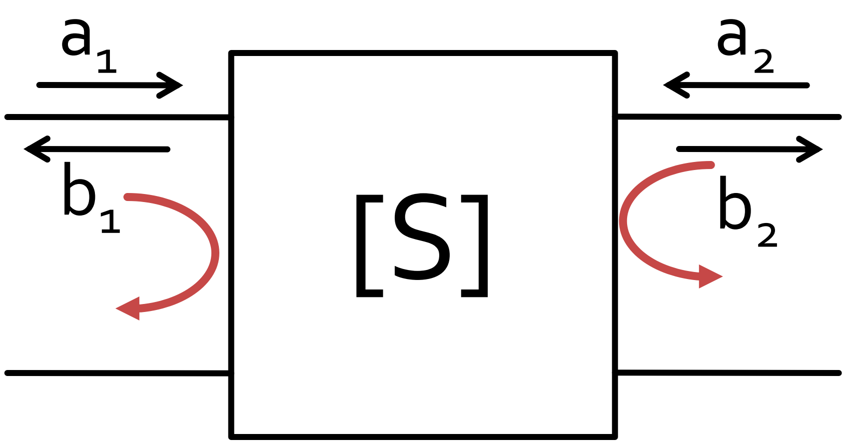
$$Z_{0i}=Z_{Ri}=R_0,\forall i$$

$$R_0=50\Omega$$

$$\left[S_p\right]\equiv[S]$$

- **they coincide!!!**

Scattering matrix – S

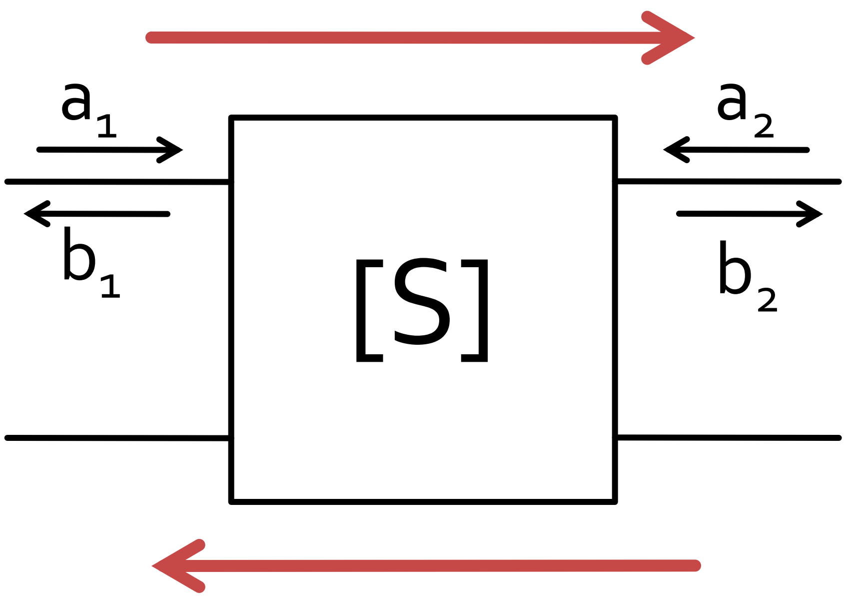


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

- S_{11} and S_{22} are reflection coefficients at ports 1 and 2 when the other port is mathed

Scattering matrix – S

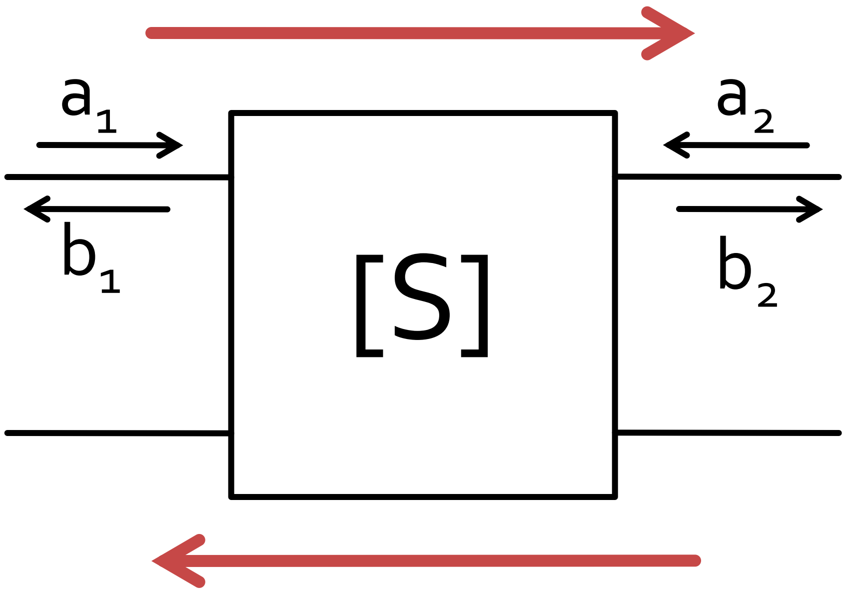


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \qquad S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude gain when the other port is matched

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ \ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Measuring S parameters - VNA

- Vector Network Analyzer

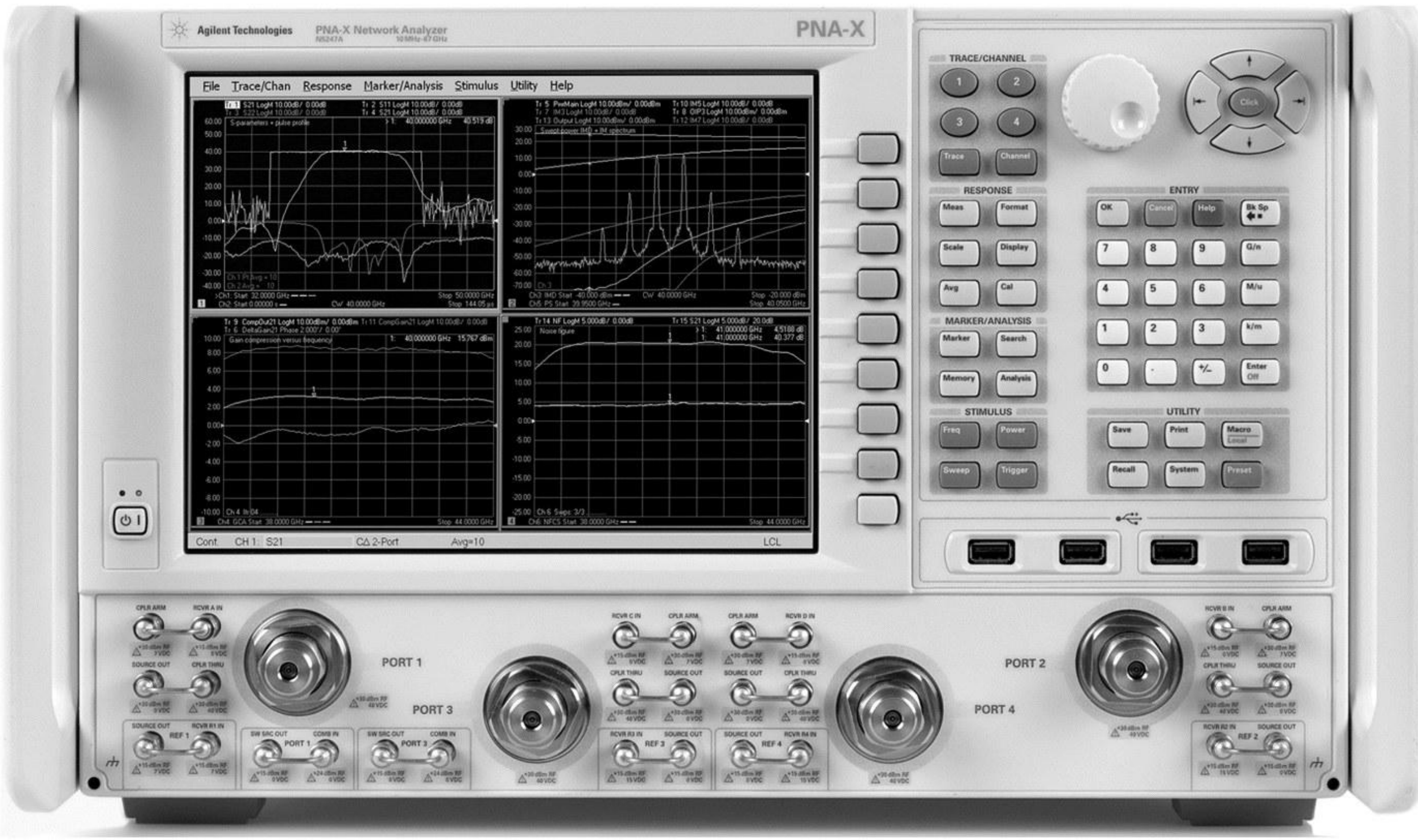


Figure 4.7
Courtesy of Agilent Technologies

Relation between two port S parameters and ABCD parameters

$$A = \sqrt{\frac{Z_{01}}{Z_{02}}}\frac{(1+S_{11}-S_{22}-\Delta S)}{2S_{21}}$$

$$B = \sqrt{Z_{01}Z_{02}}\frac{(1+S_{11}+S_{22}+\Delta S)}{2S_{21}}$$

$$C = \frac{1}{\sqrt{Z_{01}Z_{02}}}\frac{1-S_{11}-S_{22}+\Delta S}{2S_{21}}$$

$$D = \sqrt{\frac{Z_{02}}{Z_{01}}}\frac{1-S_{11}+S_{22}-\Delta S}{2S_{21}}$$

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}$$

$$S_{11} = \frac{AZ_{02}+B-CZ_{01}Z_{02}-DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{12} = \frac{2(AD-BC)\sqrt{Z_{01}Z_{02}}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{21} = \frac{2\sqrt{Z_{01}Z_{02}}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{22} = \frac{-AZ_{02}+B-CZ_{01}Z_{02}+DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

Power dividers and directional couplers

Power dividers and couplers

- Desired functionality:
  - division
  - combining
- of signal power

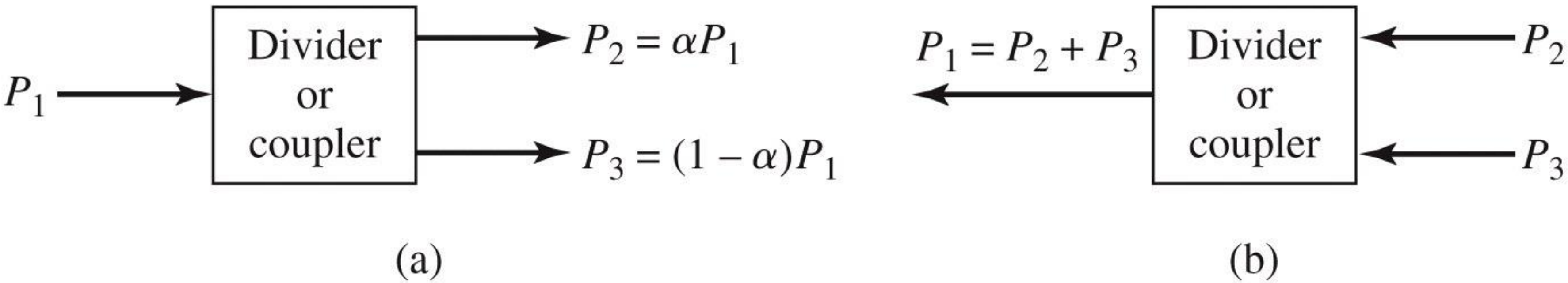


Figure 7.1

Three-Port Networks

- also known as T-Junctions
- characterized by a 3x3 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power “loss”

Three-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji}, \forall j \neq i$$

$$S_{12}=S_{21}, S_{13}=S_{31}, S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0, \forall i \qquad S_{11}=0, S_{22}=0, S_{33}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Three-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^* = 1 \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Three-Port Networks

- lossless network

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^{*} = 1$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{kj}^{*} = 0, \forall i \neq j$$

- 6 equations / 3 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{13}^{*} S_{23} = 0$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \qquad S_{12}^{*} S_{13} = 0$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad S_{23}^{*} S_{12} = 0$$

- no solution is possible

Three-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - lossless
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Nonreciprocal Three-Port Networks

- usually containing anisotropic materials, ferrites
- **nonreciprocal**, but matched at all ports and lossless $S_{ij} \neq S_{ji}$
- S matrix

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \qquad S_{21}^* S_{23} = 0$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

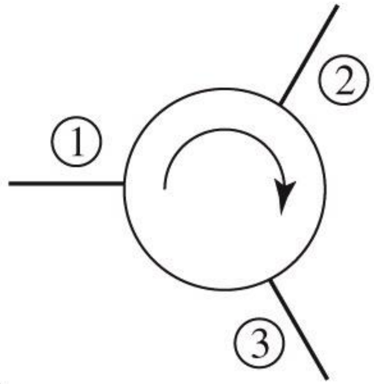
Nonreciprocal Three-Port Networks

- two possible solutions
- circulators
  - clockwise circulation

$$S_{12} = S_{23} = S_{31} = 0$$

$$|S_{21}| = |S_{32}| = |S_{13}| = 1$$

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



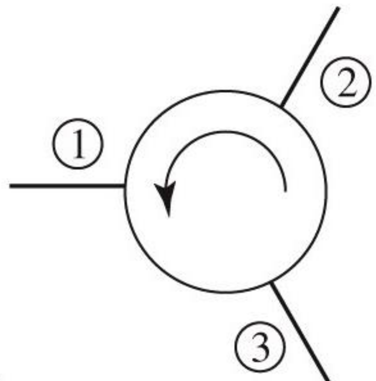
(a)

  - counterclockwise circulation

$$S_{21} = S_{32} = S_{13} = 0$$

$$|S_{12}| = |S_{23}| = |S_{31}| = 1$$

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



(b)

Mismatched Three-Port Networks

- A lossless and reciprocal three-port network can be matched only on two ports, eg. 1 and 2:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}^{*}S_{23}=0$$

$$S_{12}^{*}S_{13}+S_{23}^{*}S_{33}=0$$

$$S_{23}^{*}S_{12}+S_{33}^{*}S_{13}=0$$

$$|S_{12}|^2+|S_{13}|^2=1$$

$$|S_{12}|^2+|S_{23}|^2=1$$

$$|S_{13}|^2+|S_{23}|^2+|S_{33}|^2=1$$

$$S_{13}=S_{23}=0$$

$$|S_{13}|=|S_{23}|$$

$$|S_{12}|=|S_{33}|=1$$

Mismatched Three-Port Networks

- A lossless and reciprocal three-port network

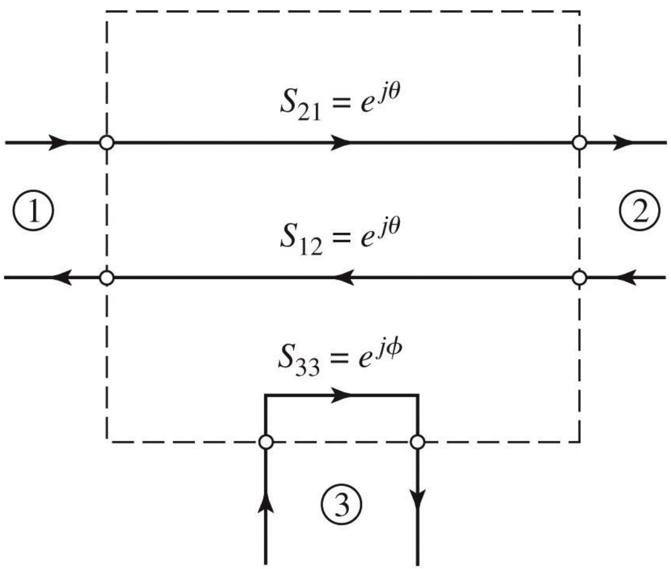
$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}=S_{23}=0 \qquad |S_{12}|=|S_{33}|=1$$

$$S_{12}=e^{j\theta}$$

$$S_{33}=e^{j\varphi}$$

$$[S]=\begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



- A lossless and reciprocal three-port network **degenerates** into two separate components:
  - a matched two-port **line**
  - a totally **mismatched one-port**:

Four-Port Networks

- characterized by a 4x4 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power "loss"

Four-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji}, \forall j \neq i$$

$$S_{12}=S_{21}, S_{13}=S_{31}, S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0, \forall i \qquad S_{11}=0, S_{22}=0, S_{33}=0, S_{44}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Four-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^* = 1 \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Four-Port Networks

$$S_{13}^* \cdot S_{23} + S_{14}^* \cdot S_{24} = 0 \quad / \cdot S_{24}^*$$
$$S_{14}^* \cdot S_{13} + S_{24}^* \cdot S_{23} = 0 \quad / \cdot S_{13}^*$$
$$S_{14}^* \cdot \left(|S_{13}|^2 - |S_{24}|^2 \right) = 0$$

$$S_{12}^* \cdot S_{23} + S_{14}^* \cdot S_{34} = 0 \quad / \cdot S_{12}$$
$$S_{14}^* \cdot S_{12} + S_{34}^* \cdot S_{23} = 0 \quad / \cdot S_{34}^*$$
$$S_{23} \cdot \left(|S_{12}|^2 - |S_{34}|^2 \right) = 0$$

- one solution: $S_{14} = S_{23} = 0$
- resulting coupler is directional

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$
$$|S_{12}|^2 + |S_{24}|^2 = 1$$
$$|S_{13}|^2 + |S_{34}|^2 = 1$$
$$|S_{24}|^2 + |S_{34}|^2 = 1$$

$$|S_{13}| = |S_{24}|$$
$$|S_{12}| = |S_{34}|$$

Four-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}|=|S_{34}|=\alpha \qquad |S_{13}|=|S_{24}|=\beta$$

β – voltage coupling coefficient

- We can choose the phase reference

$$S_{12}=S_{34}=\alpha \qquad S_{13}=\beta\cdot e^{j\theta} \qquad S_{24}=\beta\cdot e^{j\phi}$$

$$S_{12}^{*}\cdot S_{13}+S_{24}^{*}\cdot S_{34}=0 \quad \rightarrow \quad \theta+\phi=\pi\pm 2\cdot n\cdot\pi$$

$$|S_{12}|^{2}+|S_{24}|^{2}=1 \quad \rightarrow \quad \alpha^{2}+\beta^{2}=1$$

- The other possible solution for previous equations offer either essentially the same result (with a different phase reference) or the degenerate case (2 separate two port networks side by side)

$$S_{14}^{*}\cdot\left(|S_{13}|^{2}-|S_{24}|^{2}\right)=0 \qquad S_{23}\cdot\left(|S_{12}|^{2}-|S_{34}|^{2}\right)=0$$

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always** **directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Directional Coupler

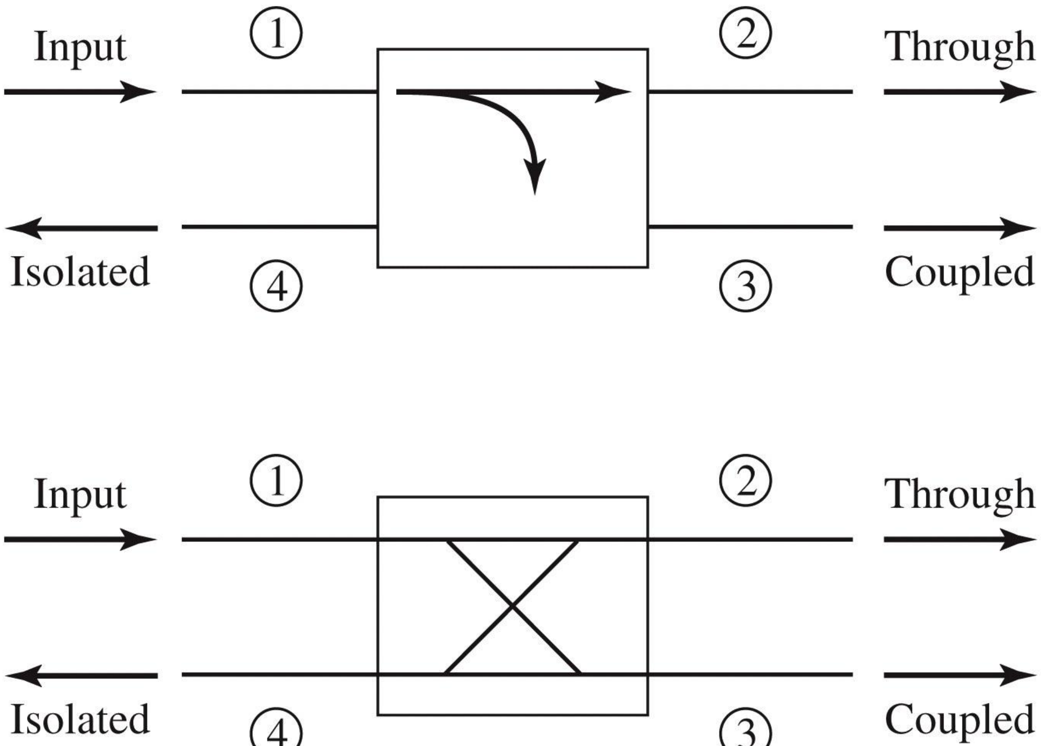


Figure 7.4

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivity

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\mathrm{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\mathrm{dB}]$$

$$I = D + C\,,\ \ dB$$

Directional Couplers

Laboratory no. 2

Directional Coupler

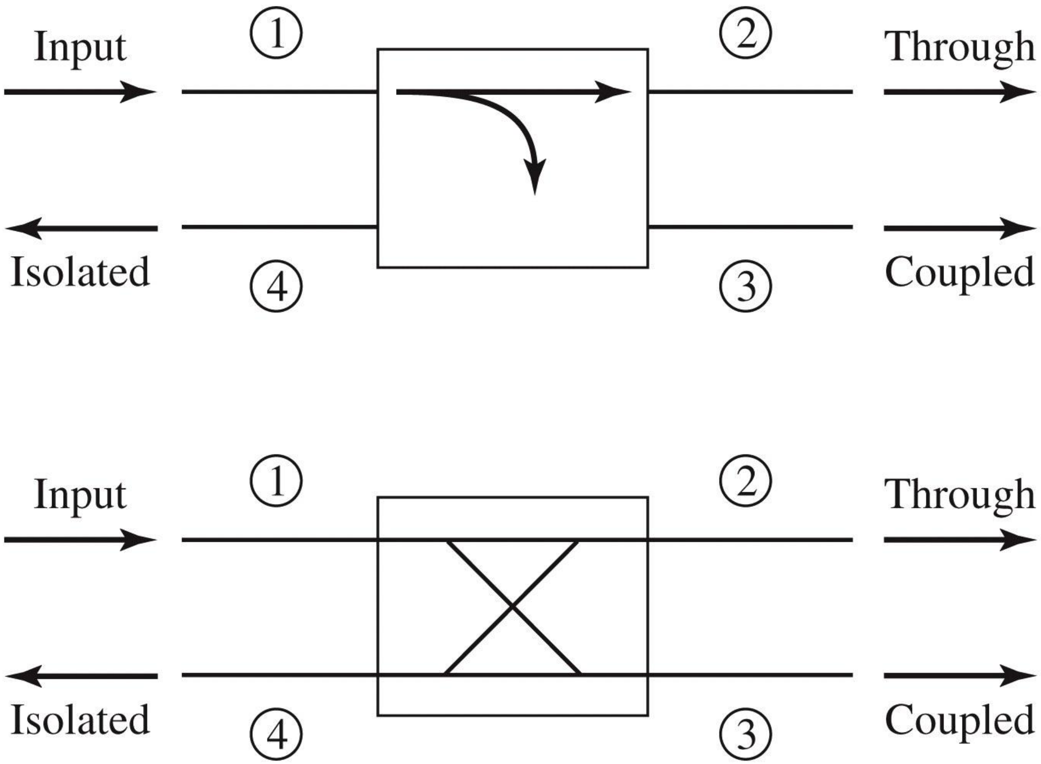


Figure 7.4

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Cuplaj

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivitate

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\mathrm{dB}]$$

Izolare

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\mathrm{dB}]$$

$$I = D + C\,,\ \ dB$$

Quadrature coupler

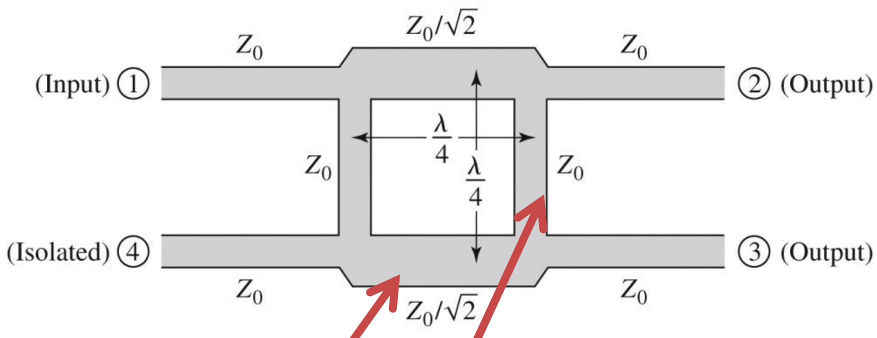


Figure 7.21

$$y_2^2 = 1 + y_1^2$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$C[\mathrm{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}$$

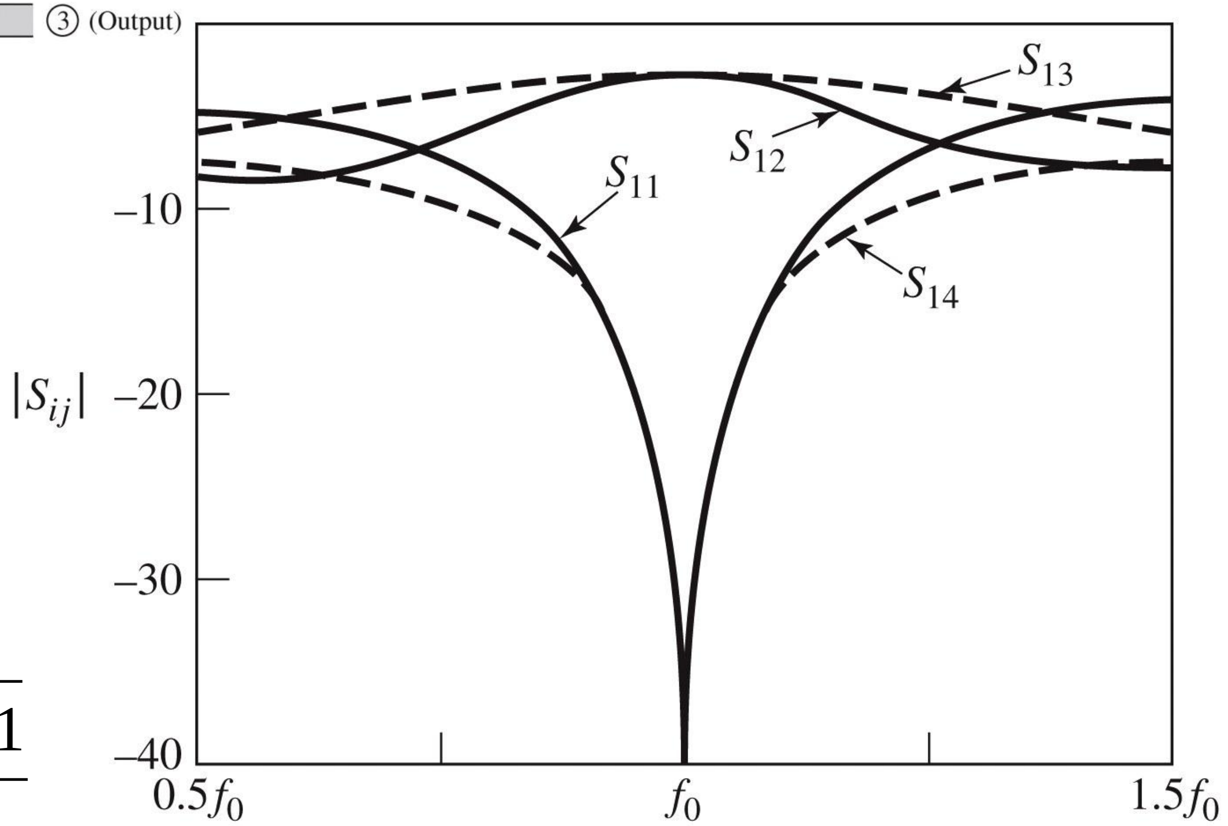
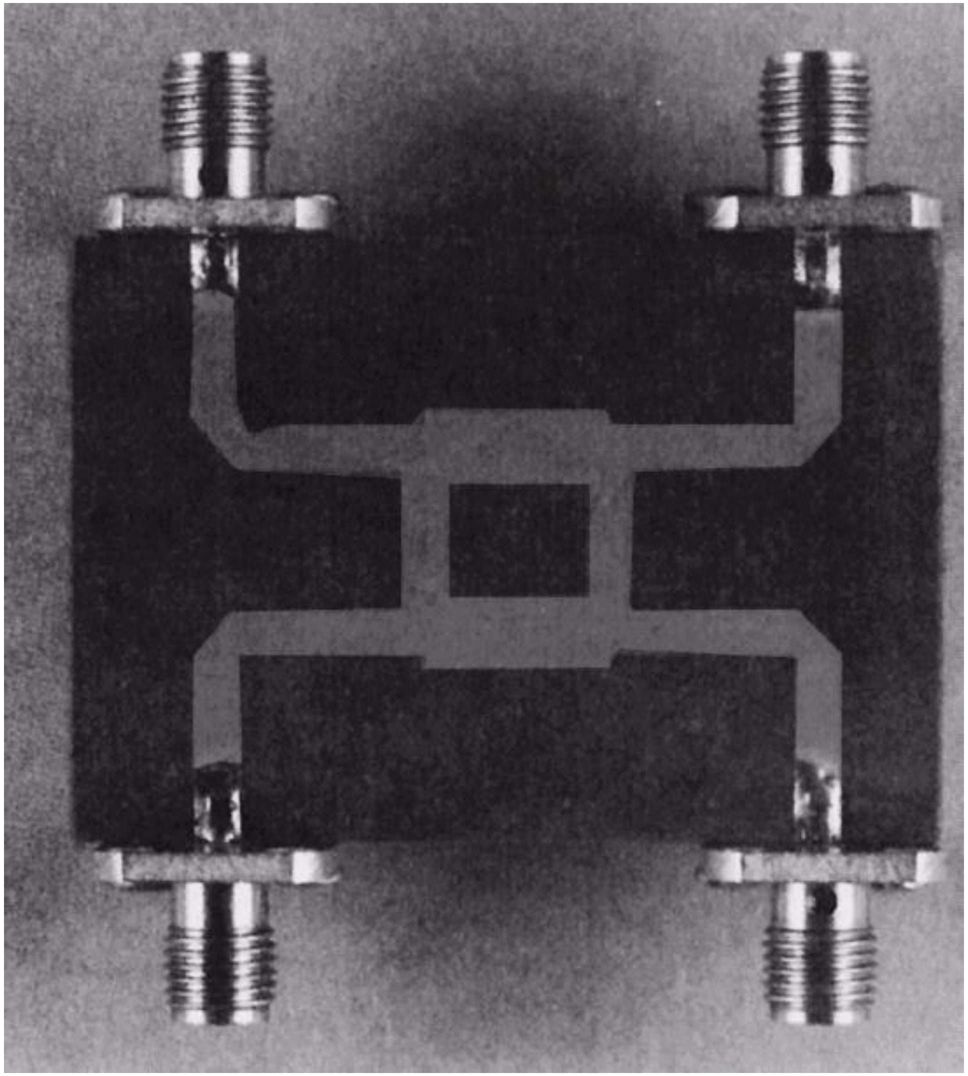
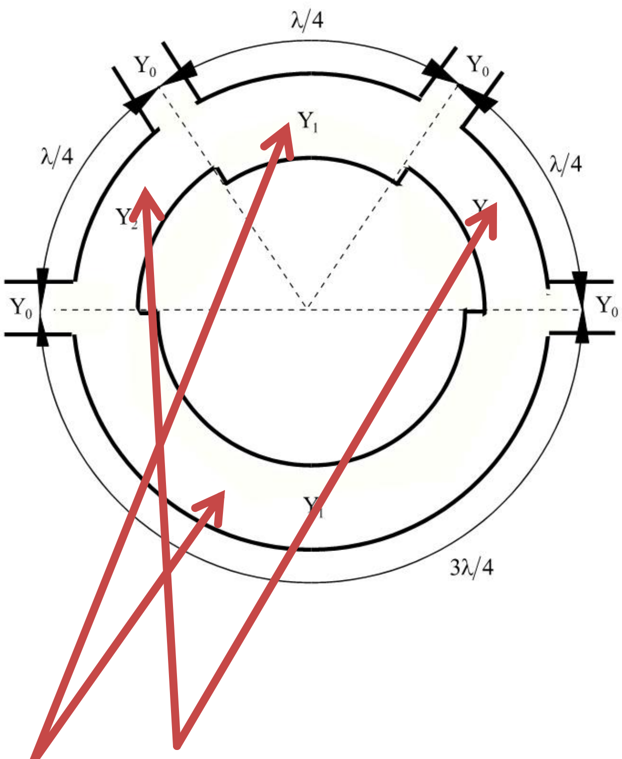


Figure 7.25

Quadrature coupler



Ring coupler



$$y_1^2 + y_2^2 = 1$$

$$|\beta| = y_1$$

$$C\,[\text{dB}] = -20 \cdot \log_{10}(y_1)$$

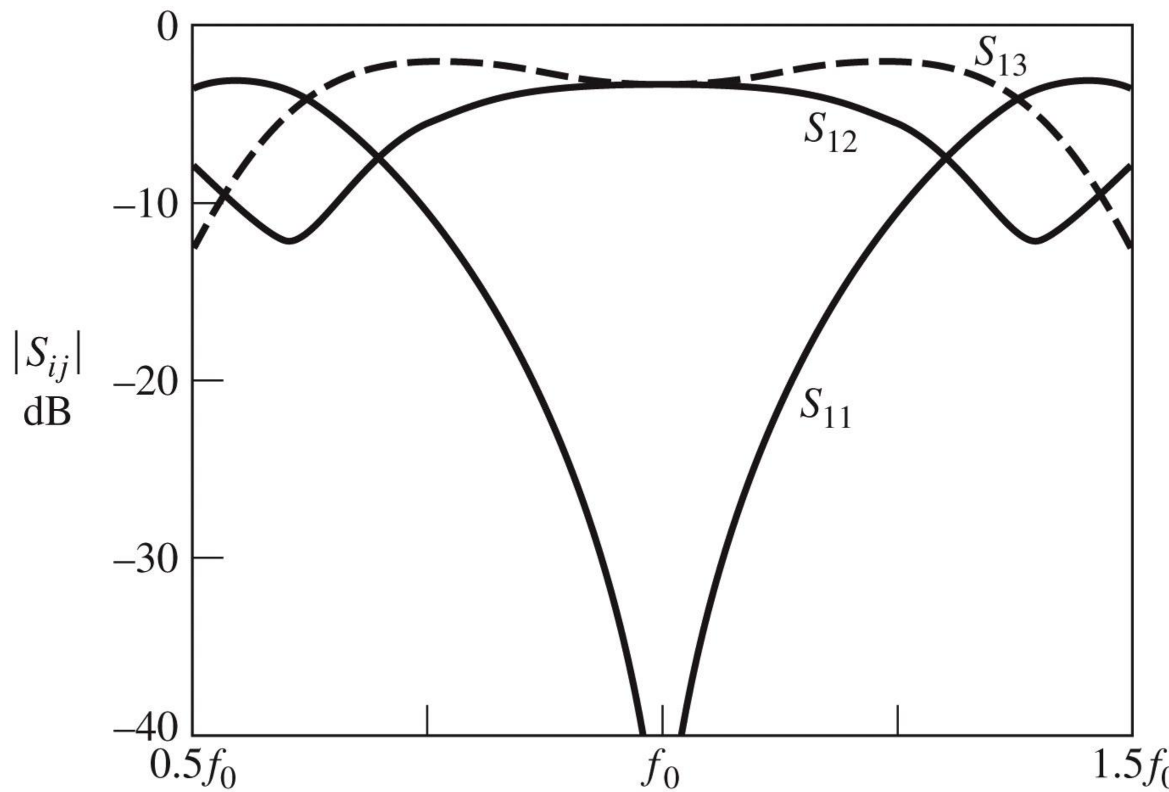


Figure 7.46

Ring coupler

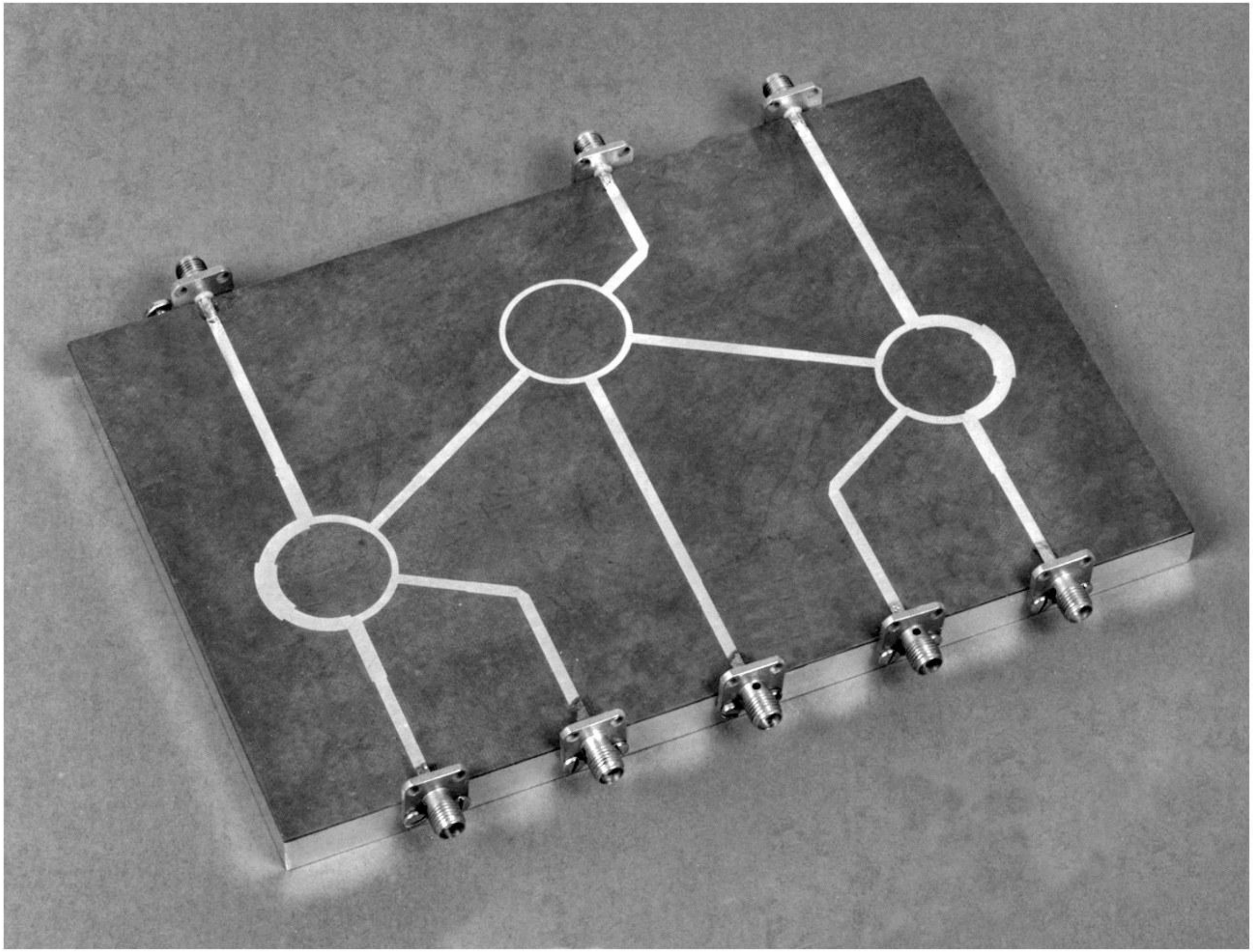
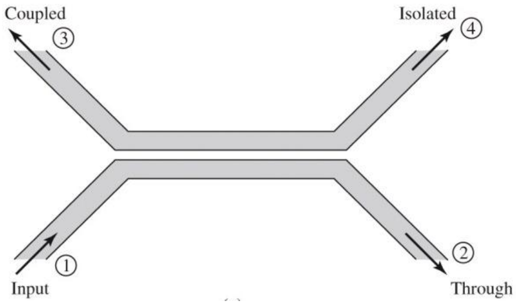


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled line coupler



$$Z_{ce} Z_{co} = Z_0^2$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$C\,[\mathrm{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)$$

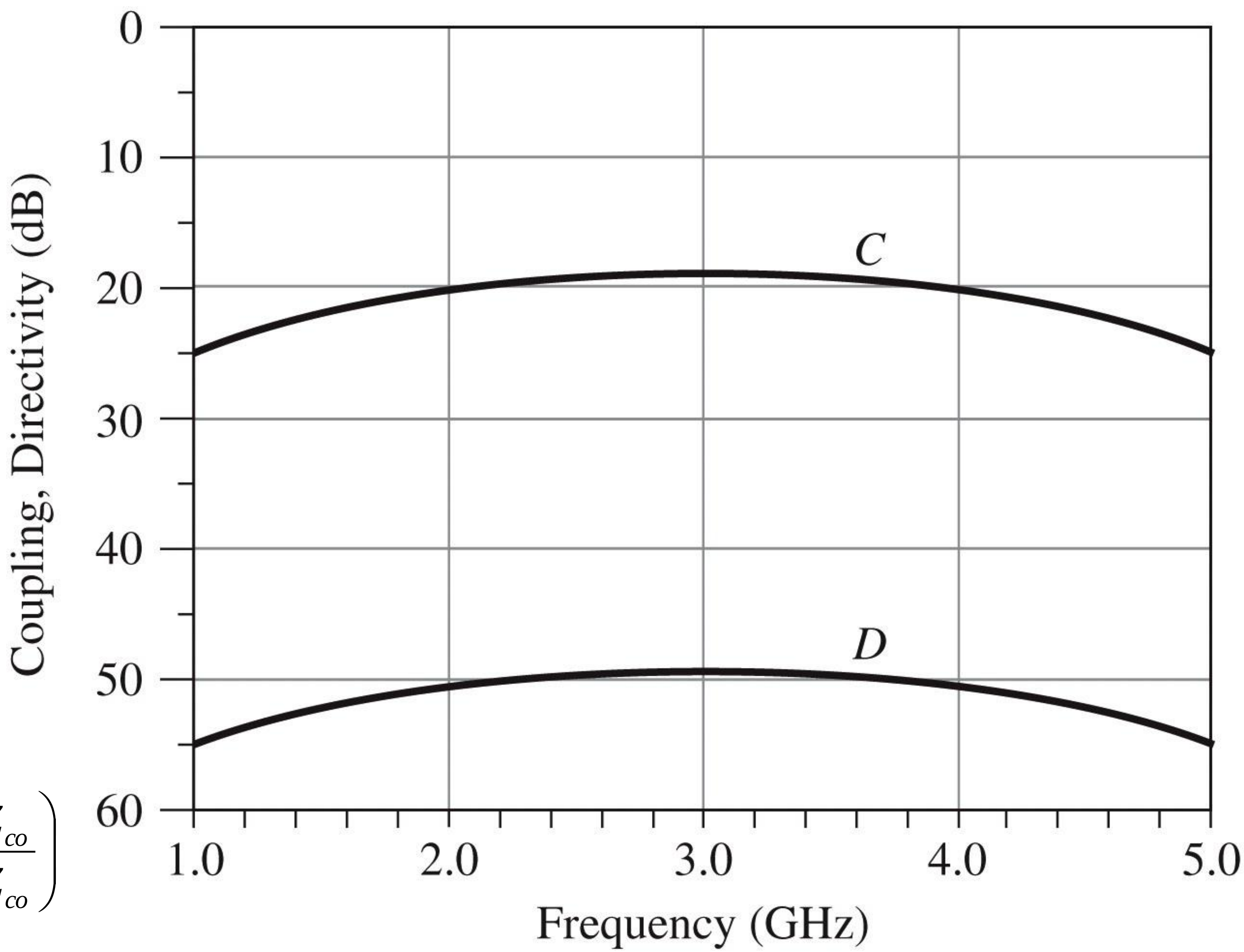
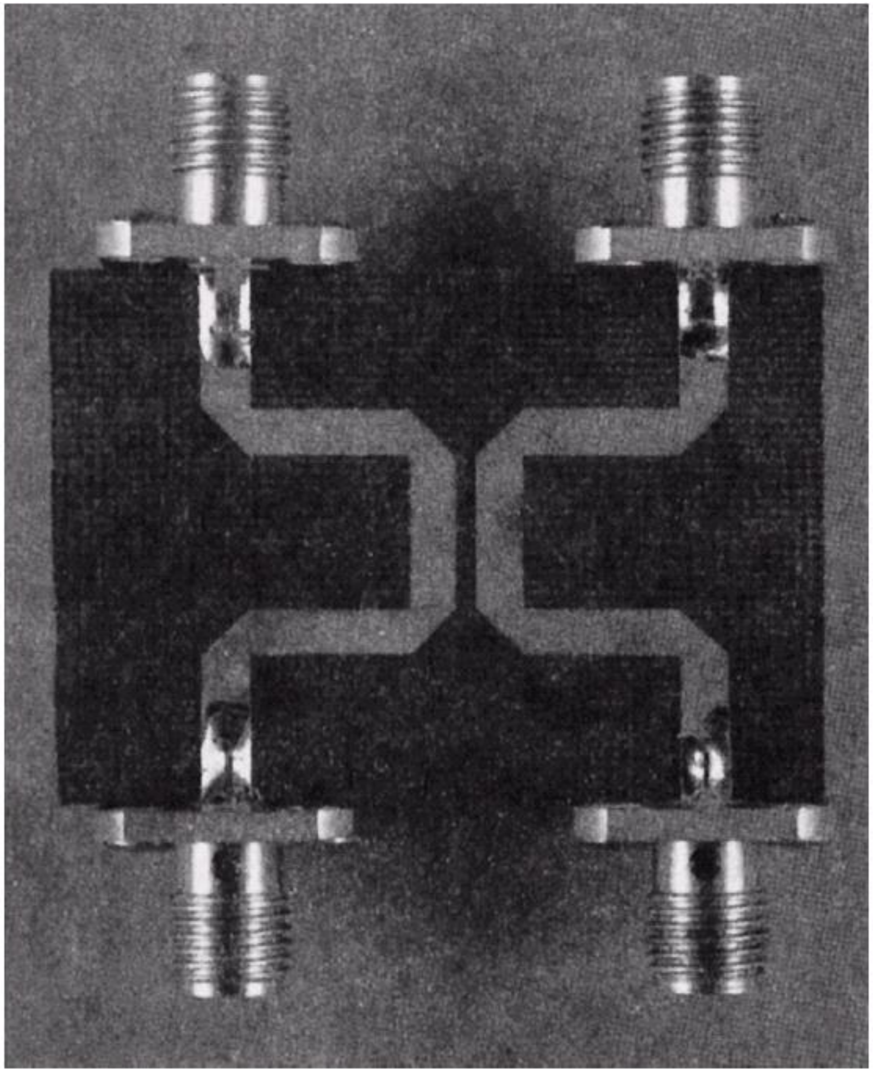


Figure 7.34

Coupled line coupler



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